NEW

PRINCIPLES

OF

Linear Perſpective:

OR THE

ART of DESIGNING

On a PLANE,

THE

REPRESENTATIONS

OF

ALL SORTS OF OBJECTS,

IN

A more General and Simple METHOD than
has been hitherto done.

The THIRD EDITION.

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PREFACE.

COnſidering how few, and how ſimple the Principles are, upon which the whole Art of PERSPECTIVE *depends, and withal how uſeful, nay how abſolutely neceſſary this Art is to all ſorts of Deſigning; I have often wonder'd, that it has ſtill been left in ſo low a degree of Perfection, as it is found to be, in the Books that have been hitherto wrote upon it. Some of thoſe Books indeed are very voluminous: but then they are made ſo, only by long and tedious Diſcourſes, explaining of common things; or by a great number of Examples, which indeed do make ſome of theſe Books valuable, by the great Variety of curious Cuts that are in them; but do not at all inſtruct the Reader, by any Improvements made in the Art itſelf. For it ſeems that thoſe, who have hitherto treated of this Subject, have been more converſant in the Practice of Deſigning, than in the Principles of Geometry; and therefore when, in their Practice, the Occaſions that have offer'd, have put them upon inventing particular Expedients, they have thought them to be worth communicating to the Public, as Improvements in this Art; but they have not been able to produce any real Improvements in it, for want of a ſufficient Fund of Geometry, that might have enabled them to render the Principles of it more univerſal, and more convenient for Practice. In this Book I have endeavour'd to do this; and have done my utmoſt* *to render the Principles of the Art as general, and as univerſal as may be, and to deviſe ſuch Conſtructions, as might be the moſt ſimple and uſeful in Practice.*

In order to this, I found it abſolutely neceſſary to conſider this Subject entirely anew, as if it had never been treated of before; the Principles of the old Perſpective being ſo narrow, and ſo confined, that they could be of no uſe in my Deſign: And I was forced to invent new Terms of Art, thoſe already in uſe being ſo peculiarly adapted to the imperfect Notions that have hitherto been had of this Art, that I could make no uſe of them in explaining thoſe general Principles I intended to eſtabliſh. The Term of Horizontal Line, for inſtance, is apt to confine the Notions of a Learner to the Plane of the Horizon, and to make him imagine, that that Plane enjoys ſome particular Privileges, which make the Figures in it more eaſy and more convenient to be deſcribed, by the means of that Horizontal Line, than the Figures in any other Plane; as if all other Planes might not as conveniently be handled, by finding other Lines of the ſame nature belonging to them. But in this Book I make no difference between the Plane of the Horizon, and any other Plane whatſoever; for ſince Planes, as Planes, are alike in Geometry, it is moſt proper to conſider them as ſo, and to explain their Properties in general, leaving the Artiſt himſelf to apply them in particular Caſes, as Occaſion requires.

My Deſign in this Book, is not to trouble the Reader with a Multitude of Examples, nor to deſcend to a perfect Explanation of any particu-

lar Caſes; but to explain the general Principles of Perſpective: *Which if I have been ſo happy as to have done, in ſuch a manner as may be intelligible to the Reader; I don't doubt but he will eaſily be inclined to pardon my Shortneſs: For he will find much more pleaſure in obſerving how extenſive theſe Principles are, by applying them to particular Caſes which he himſelf ſhall deviſe, while he exerciſes himſelf in this Art, than he would do in reading the tedious Explanations of Examples deviſed by another.*

I find that many People object to the firſt Edition that I gave of theſe Principles, in the little Book entituled, Linear Perſpective, &c. *becauſe they ſee no Examples in it, no curious Deſcriptions of Figures, which other Books of* Perſpective *are commonly ſo full of; and ſeeing nothing in it but ſimple Geometrical Schemes, they apprehend it to be dry and unentertaining, and ſo are loth to give themſelves the trouble to read it. To ſatisfy theſe nice Perſons in ſome meaſure, I have made the Schemes in this Book ſomething more ornamental, that they may have ſome viſible Proofs of the vaſt Advantages theſe Principles have over the common Rules of* Perſpective, *by ſeeing what ſimple Conſtructions, and how few Lines are neceſſary to deſcribe ſeveral Subjects, which in the common Method would require an infinite Labour, and a vaſt Confuſion of Lines. It would have been eaſy to have multiplied Examples, and to have enlarged upon ſeveral things that I have only given Hints of, which may eaſily be purſued by thoſe who have made themſelves Maſters of theſe Principles. Perhaps ſome*

People would have been better pleaſed with my Book, if I had done this: but I muſt take the freedom to tell them, that tho' it might have amuſed their Fancy ſomething more by this means, it would not have been more inſtructive to them: for the true and beſt way of learning any Art, is not to ſee a great many Examples done by another Perſon; but to poſſeſs ones ſelf firſt of the Principles of it, and then to make them familiar, by exerciſing ones ſelf in the Practice. For it is Practice alone, that makes a Man perfect in any thing.

The Reader, who underſtands nothing of the Elements of Geometry, can hardly hope to be much the better for this Book, if he reads it without the Aſſiſtance of a Maſter; but I have endeavour'd to make every thing ſo plain, that a very little Skill in Geometry may be ſufficient to enable one to read this Book by himſelf. And upon this occaſion I would adviſe all my Readers, who deſire to make themſelves Maſters of this Subject, not to be contented with the Schemes they find here; but upon every Occaſion to draw new ones of their own, in all the Variety of Circumſtances they can think of. This will take up a little more Time at firſt; but in a little while they will find the vaſt Benefit of it, by the extenſive Notions it will give them of the Nature of theſe Principles.

The Art of Perſpective *is neceſſary to all Arts, where there is any occaſion for Deſigning; as* Architecture, Fortification, Carving, *and generally all the Mechanical Arts; but it is more particularly neceſſary to the Art of* Painting, *which can do* *do nothing without it. A Figure in a Picture, which is not drawn according to the Rules of* Perſpective, *does not repreſent what is intended, but ſomething elſe. So that it ſeems to me, that a Picture which is faulty in this particular, is as blameable, or more ſo, than any Compoſition in Writing, which is faulty in point of Orthography, or Grammar. It is generally thought very ridiculous to pretend to write an Heroic Poem, or a fine Diſcourſe upon any Subject, without underſtanding the Propriety of the Language wrote in; and to me it ſeems no leſs ridiculous for one to pretend to make a good Picture without underſtanding* Perſpective : *Yet how many Pictures are there to be ſeen, that are highly valuable in other reſpects, and yet are entirely faulty in this point? Indeed this Fault is ſo very general, that I cannot remember that I ever have ſeen a Picture, that has been entirely without it; and what is the more to be lamented, the Greateſt Maſters have been the moſt guilty of it. Thoſe Examples make it to be the leſs regarded; but the Fault is not the leſs, but the more to be lamented, and deſerves the more Care in preventing it for the future. The great Occaſion of this Fault, is certainly the wrong Method that generally is uſed in the Education of Perſons to this Art: For the Young People are generally put immediately to Drawing, and when they have acquired a Facility in that, they are put to Colouring. And theſe things they learn by rote, and by Practice only; but are not at all inſtructed in any Rules of Art. By which means when they come to make any Deſigns of their own, tho' they are very*

very expert at drawing out, and colouring every thing that offers itſelf to their Fancy; yet for want of being inſtructed in the ſtrict Rules of Art, they don't know how to govern their Inventions with Judgment, and become guilty of ſo many groſs Miſtakes, which prevent themſelves, as well as others, from finding that Satisfaction, they otherwiſe would do, in their Performances. To correct this for the future, I would recommend it to the Maſters of the Art of Painting, *to conſider if it would not be neceſſary to eſtabliſh a better Method for the Education of their Scholars, and to begin their Inſtructions with the Technical Parts of* Painting, *before they let them looſe to follow the Inventions of their own uncultivated Imaginations.*

The Art of Painting, *taken in its full Extent, conſiſts of two Parts; the Inventive, and the Executive. The Inventive Part is common with* Poetry, *and belongs more properly and immediately to the Original Deſign (which it invents and diſpoſes in the moſt proper and agreeable manner) than to the Picture, which is only a Copy of that Deſign already formed in the Imagination of the Artiſt. The Perfection of this Art of* Painting *depends upon the thorough Knowledge the Artiſt has of all the Parts of his Subject; and the Beauty of it conſiſts in the happy Choice and Diſpoſition that he makes of it: And it is in this that the Genius of the Artiſt diſcovers and ſhews it ſelf, while he indulges and humours his Fancy, which here is not confined. But the other, the Executive Part of* Painting, *is wholly confined, and ſtrictly tied to the Rules of Art,* *which cannot be diſpenſed with upon any account; and therefore in this the Artiſt ought to govern himſelf intirely by the Rules of Art, not to take any liberties whatſoever. For any thing that is not truly drawn according to the Rules of* Perſpective, *or not truly colour'd, or truly ſhaded, does not appear to be what the Artiſt intended, but ſomething elſe. Wherefore if at any time the Artiſt happens to imagine, that his Picture would look the better, if he ſhould ſwerve a little from theſe Rules, he may aſſure himſelf, that the Fault belongs to his* Original *Deſign, and not to the Strictneſs of the Rules; for what is perfectly agreeable and juſt in the real* Original *Objects themſelves, can never appear defective in a Picture, where thoſe Objects are exactly copied.*

Therefore to offer a ſhort Hint of the Thoughts I have ſome time had upon the Method which ought to be follow'd in inſtructing a Scholar in the Executive Part of Painting; *I would firſt have him learn the moſt common Effections of Practical Geometry, and the firſt Elements of Plain Geometry, and common Arithmetic. When he is ſufficiently perfect in theſe, I would have him learn* Perſpective. *And when he has made ſome progreſs in this, ſo as to have prepared his Judgment with the right Notions of the Alterations that Figures muſt undergo, when they come to be drawn on a Flat, he may then be put to Drawing by View, and be exerciſed in this along with* Perſpective, *till he comes to be ſufficiently perfect in both. Nothing ought to be more familiar to a* Painter *than* Perſpective; *for it is the only*

only thing that can make the Judgment correct, and will help the Fancy to invent with ten times the eaſe that it could do without it. For the Colouring; before the Young Artiſt is employ'd in copying of Pictures, where there are great Variety of Colours *to be imitated, it would be well that he ſhould be inſtructed in the Theory of the* Colours*; that he ſhould learn to know their particular Properties, their different Relations, and the various Effects that are produced by their Mixture; and that he ſhould be made well acquainted with the Nature of the ſeveral material Colours that are uſed in* Painting. *Theſe things ought to be learnt in a regular Method; and the Artiſt ought not to depend intirely upon the ſeveral indigeſted Obſervations, that may occur to him in Practice. And to this laſt Purpoſe of Colouring, I cannot help thinking, that the Theory I have endeavour'd to explain in the* Appendix, *from Sir* Iſaac Newton, *may be of very great uſe to Learners. There may be regular Methods alſo invented for teaching the Doctrine of Light and Shadow; and other Particulars relating to the Practical Part of* Painting, *may be improved and digeſted into proper Methods for inſtructing the Young Artiſt. But I only hint at theſe things, recommending them to the Maſters of the Art to reflect and improve upon.*

The Book it ſelf is ſo ſhort, that I need not detain the Reader any longer in the Preface, *by giving him a more particular Account of what he may expect to find in it.*

LINEAR PERSPECTIVE.

PART I.

DEFINITIONS.

DEFIN. I.

LINEAR PERSPECTIVE is the Art of deſcribing exactly, on a Plane Surface, the Repreſentations of any given Objects.

TO have a compleat and clear Notion of the Principles of this Art, let the Reader conſider, that a Picture drawn in the utmoſt degree of Perfection, and placed in a proper Poſition, ought ſo to appear to the Spectator, that he ſhould not be able to diſtinguiſh what is there repreſented, from the real original Objects actually placed where they are repreſented to be. In order to produce this Effect, it is neceſſary that the Rays of Light ought to come from the ſeveral Parts of the Picture to the Spectator's Eye, with all the ſame Circumſtances of Direction, Strength of Light and Shadow, and Colour, as they would do from the correſponding Parts of the real Objects ſeen in their proper Places. Thus (*Fig.* 1.) ſuppoſing the Spectator, whoſe Eye is O, to be looking on the Picture of a Cube *a b c d e*, and ABCDE to be the real original Cube, actually placed where it ſeems to him to be; the Light from any Point *a* of the Picture ought to come to the Spectator's Eye O by the Ray *a* O, in the ſame Direction, with the ſame Colour, and with the ſame Strength of Light and Shadow, as it would do from the correſponding Point A of the Original Cube, by the Ray A O. The three Circumſtances juſt mentioned make the executive Part of the Art of Painting to conſiſt of three Parts, *viz.* Drawing, (which relates wholly to the Poſition, and conſequently to the Shapes of the Figures on the Picture; and which when it is done exactly by Mathematical Rules, and not by an acquired Habit of the Hand and Eye, is *Perſpective*, the Subject of the preſent Diſcourſe;) Colouring, and the Art of Light and Shadow, which the *Italians* call the *Chiaroſcuro.* The Principles of all theſe Parts of Painting are to be drawn from this general Conſideration, and particularly thoſe of *Perſpective.* Wherefore in the Demonſtrations of the following Propoſitions in this Book, we muſt always have recourſe to this general Foundation, by ſhewing that the Rays of Light will come in the ſame Directions from the ſeveral Parts aſſigned in the Picture, as they would do if they came from the correſponding Parts of the original Objects placed in their proper Situations.

DEFIN. II.

When Lines drawn according to a certain Law from the ſeveral Parts of any Figure, cut a Plane, and by that Cutting or Interſection deſcribe a Figure on that Plane, that Figure ſo deſcribed is called the *Projection* of the other Figure. The Lines producing that Projection, taken all together, are called the *Syſtem of Rays*. And when thoſe Rays all paſs thro' one and the ſame Point, they are called the *Cone of Rays*. And when that Point is conſider'd as the Eye of a Spectator, that Syſtem of Rays is called the *Optic Cone*.

Projectio

System of

Cone of R

Optic C

DEFIN. III.

When the Syſtem of Rays are all parallel to each other, and perpendicular to the Horizon, and the Projection is made on a Plane parallel to the Horizon, it is called the *Ichnography* of the Figure propoſed.

Ichnogr

Ichno

poſed. Thus (*Fig.* 2.) the Plane GHIK being parallel to the Horizon, and the Rays A *a*, B *b*, C *c*, *&c.* coming from the ſeveral Parts of the *Octaedron* ABC DEF, being perpendicular to it, and parallel to each other, the Projection *a b c d e* made by them is the *Ichnography* of the Figure ABCDEF.

DEFIN. IV.

When the Syſtem of Rays are parallel to each other, and to the Horizon, and the Projection is made on a Plane perpendicular to thoſe Rays, and to the hography Horizon, it is called the *Orthography* of the Figure propoſed. Thus (*Fig.* 2.) the Rays A a, B b, C c, *&c.* being parallel to the Horizon, and to each other, and the Plane G H L M, on which the Projection is made, being perpendicular to them, abcdef is the *Orthography* of the Figure ABCDEF.

Theſe are the common Definitions of the Terms *Ichnography* and *Orthography*, but we ſhall hereafter uſe them to ſignify any two Projections, that are made by Syſtems of Parallel Rays, when thoſe Syſtems are perpendicular to each other, and to the Planes on which the Projections are made, as in the preſent Figure, without having any regard to their Situation with reſpect to the Horizon.

In this kind of Projections, the Projection on any particular Point, or Line, is ſometimes called the Seat of that Point, or Line, on the Plane of the Projection. Thus *a* is the Seat of the Point A on the Plane G H I K, and a e is the Seat of the Line A F on the Plane G H L M.

DEFIN. V.

When the Projection is made by a Cone of Rays, it is called the *Schenography*. Thus (*Fig*. 1.) the Figure *a b c d e* projected on the Plane F G H I by the Rays A O, B O, C O, *&c.* coming from the ſeveral Parts of the Cube A B C D E to the Point O, is the *Schenography* of the Figure ABCDE.

Schenog

We ſhall hereafter ſhew this Projection to be the Picture of the Object A B C D E, to be ſeen by a Spectator's Eye at O.

It is alſo evident that the Shadows of Figures are this Projection, when the Light is conſider'd as a ſingle Point. Tho' in the Caſe of the Sun or Moon, that Point being at an infinite Diſtance (as to all Senſe) the Projecting Rays are parallel to each other.

DEFIN. VI.

int of Sight

The *Point of Sight* is that Point, where the Spectator's Eye ought to be placed, to look upon the Picture.

This Point is no other than the Vertex of the Optic Cone, as will be evident from *Theor.* 2. where we ſhew, that the Repreſentation of any Object is no other than its *Schenographic* Projection on the Plane of the Picture.

DEFIN. VII.

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ture &
istance

If from the Point of Sight there be drawn a Line perpendicular to the Picture, the Point where that Line cuts the Picture is called the *Center* of the Picture. And the Diſtance between that Center and the Point of Sight, is called the *Diſtance* of the Picture.

DEFIN. VIII.

rect Plane

If thro' the Point of Sight there be imagined to paſs a Plane parallel to the Picture, that Plane is called the *Directing Plane.*

DEFIN.

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DEFIN. IX.

By *Original Object* (whether it be Point, Line, Surface or Solid) we mean the real Object placed in the Situation it is repreſented to have by the Picture.

Origina Object

DEFIN. X.

By *Original Plane*, we mean the Plane wherein is ſituated any Original Point, Line, or Plane Figure.

Origina Plan

DEFIN. XI.

The Point where any Original Line (continued if need be) cuts the Picture, is called ſimply the *Interſection* of that Line.

Intersec of a line

DEFIN. XII.

The Line wherein any Original Plane cuts the Picture, is called ſimply the *Interſection* of that Original Plane.

Intersecti of Origin plane

DEFIN. XIII.

The Point where any Original Line cuts the Directing Plane is called the *Directing*

Directg Point

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Original
Line

ing Point of that Original Line. And a Line drawn thro' that directing Point, and the Point of Sight, is call'd the *Director* of that Original Line.

DEFIN. XIV.

Directing
Line

The Line wherein any Original Plane cuts the Directing Plane, is call'd the *Directing Line* of that Original Plane.

DEFIN. XV.

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Original
ine

A Line drawn thro' the Point of Sight parallel to any Original Line, is called ſimply the *Parallel* of that Original Line.

DEFIN. XVI.

rallel
Original
Plane

A Plane paſſing thro' the Point of Sight parallel to any Original Plane, is called ſimply the *Parallel* of that Original Plane.

DEFIN. XVII.

anish: Point
a Line

The Point where the Parallel of any Original Line cuts the Picture, is call'd the *Vaniſhing Point* of that Line. And the

Diſtance between that Vaniſhing Point and the Point of Sight, is called ſimply the *Diſtance* of that Vaniſhing Point.

Dista of Vani Point

DEFIN. XVIII.

The Line wherein the Parallel of any Original Plane cuts the Picture, is called the *Vaniſhing Line* of that Plane. And if from the Point of Sight there be drawn a Line cutting that Vaniſhing Line at Right Angles, the Point where that Vaniſhing Line is ſo cut, is called the *Center* of it. And the Diſtance between that Center and the Point of Sight, is called ſimply the *Diſtance* of that Vaniſhing Line.

Vanish line of Plane

Center Vanij P.

Distance of Vanis Line

DEFIN. XIX.

The Repreſentation of any Figure is called the *Projection* of that Figure.

Projection a Figure

In order to comprehend the Senſe of theſe Definitions more fully, let the Reader imagine the Plane A B C (*Fig.* 3.) to be the Surface of the Picture, O to be the Point of Sight (*Def.* 6.) the Plane O D E to be the Directing Plane parallel to the Picture (*Def.* 8.) F G to be an Original Line (*Def.* 9.) in the Original Plane F G H (*Def.* 10.)

cutting the Picture in B I, and the Directing Plane in D E. And let O A C be a Plane parallel to the Original Plane F G H, and cutting the Picture in the Line A C; and let the Line O V be parallel to the Original Line F G, and cut the Picture in V. And let F G cut the Picture in B, and the Directing Plane in D. Theſe things being ſuppoſed, B is the Interſection of the Original Line F G (*Def.* 11.) D its Directing Point (*Def.* 13.) and V its Vaniſhing Point (*Def.* 17.) and O D its Director (*Def.* 13.) and O V is the Diſtance of the Vaniſhing Point V (*Def.* 17.) B I is the Interſection (*Def.* 12.) D E the Directing Line (*Def.* 14.) O A C the Parallel (*Def.* 16.) and A C the Vaniſhing Line (*Def.* 18.) of the Original Plane F G H. And if there be drawn O S cutting the Vaniſhing Line A C at Right Angles in S, that Point S will be the Center (*ib.*) and S O will be the Diſtance (*ib.*) of the Vaniſhing Line A C.

AXIOM. I.

The common Interſection of two Planes is a ſtraight Line.

AXIOM. II.

If two ſtraight Lines meet in a Point, or are parallel to one another, there may be a Plane paſſing thro' them both.

AXIOM III.

If three ſtraight Lines cut one another, or if two of them being parallel, are both cut by the third, they will all three be in the ſame Plane; that is, a Plane paſſing thro' any two of them will alſo paſs thro' the third.

AXIOM. IV.

Every Point in any ſtraight Line, is in any Plane that Line is in.

LEMMA I.

If B S O (Fig. 4.) *and* A E B D *be two Planes, cutting each other in the Line* A S B, *and from any Point* O *of one of the Planes be drawn two Lines* O S *and* O C *cutting the Line* A B, *and the other Plane* A E B D *at Right Angles in* S *and* C, *and there be drawn* C S; *that Line* C S *will be perpendicular to* A S B.

This follows from *Prop.* 11. *Lib.* 11. *Elem.*

THEOREM I.

A Line drawn from the Center of the Picture to the Center of a Vaniſhing Line, is perpendicular to that Vaniſhing Line.

DEMONSTRATION,

Imagine AEBD (*Fig.* 4.) to be the Plane of the Picture, O to be the Point of Sight, and O S B to be a Parallel Plane producing the Vaniſhing Line A S B by its Interſection with the Picture; and let S be the Center of that Vaniſhing Line, and C be the Center of the Picture. Then having drawn O S and O C, draw C S. O S is perpendicular to A S (by *Def.* 18.) and O C is perpendicular to the Plane A E B D (by *Def.* 7.) Therefore C S is perpendicular to A S (by *Lem.* 1.) Which was to be proved.

COROL. The Diſtance O S of any Vaniſhing Line A S B, is the Hypothenuſe of a Right-angled Triangle, whoſe Legs are the Diſtance of the Picture O C, and the Diſtance C S between the Center of that Vaniſhing Line and the Center of the Picture.

THEOR. II.

The Perſpective Repreſentation, or Projection, of any Object, is the ſame as the Ichnographic Projection of it on the Plane of the Picture, the Point of Sight being the Vertex of the Optic Cone.

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DEMONSTRATION.

For by the Explanation of the Principles of the Art of Painting, in *Def.* 1. ſince the Light muſt come to the Spectator's Eye O (*Fig.* 1.) in the ſame Direction from any Point *a* of the Projection, as it would do from the corresponding Point A of the original Object, it is evident that the Rays *a O* and A O are in one and the ſame ſtraight Line. Whence it is evident, that the Projection *a* is the Interſection of the Picture with the Ray A O, and the whole Projection *abcde* is the Schenographic Projection of the original Figure A B C D E made by the Optic Cone O A B C D E, whoſe Vertex is the Point of Sight O. Which was to be proved.

COROL. 1. The Projection of a ſtraight Line is a ſtraight Line. For the Optic Cone ODE, which produces the Projection *de* of any Line DE, is the Surface of a Plane Triangle ODE, all the Rays going to O from the ſeveral Points of the Line DE being in the Plane paſſing thro' the Lines DO and EO. Therefore *de* is the Interſection of the Picture with the Plane Triangle ODE, and conſequently is a ſtraight Line (by *Ax.* 1.)

COROL. 2. The Original of a Projection may be any Object, that will produce the ſame Cone of Rays. Thus the Original of the Projection *de*, may be any Line d e, which produces the Optic Cone ODE, as well as the Line DE.

This being ſo, it may reaſonably be ask'd whence it comes that Figures drawn on a Picture appear to be what they are deſigned to repreſent. The Reaſon is, becauſe the Mind has got a Habit of judging Objects, that are ſo and ſo related, have ſuch and ſuch Colours, and are ſo and ſo enlightned and ſhaded, to be of ſuch and ſuch a Shape, and to be ſo and ſo ſituated. Theſe Circumſtances are all of them neceſſary to make a Picture compleat, tho' the ſimple Drawing is ſometimes almoſt ſufficient, upon acccount of the Relation of the Parts; as in a Pavement, where all the Stones appear to be ſquare, tho' they are repreſented by very irregular Figures. I ſay, it is the Relation of the Parts which produces this Effect; for the Repreſentation of any one of the ſingle Squares would hardly appear to be ſquare, if there were no other Objects to biaſs the Judgment by their Relation to it.

THEOR. III.

The Projection of a ſtraight Line not parallel to the Picture, paſſes thro' both its Interſection and Vaniſhing Point.

DEMONSTRATION.

All other things in *Fig.* 3. being underſtood (as above at the End of the Definitions) let *fg* be the Projection of the Original Line FG, FO, and GO being the Rays which produce the Projections *f* and *g* of the Points F and G (See *Th.* 2.) By *Theor.* 2. *fg* is the Interſection of the Picture with the Plane of the Triangle OFG. But all the Line FGB is in that Plane; and conſequently the Interſection B. Wherefore *fg* continued muſt paſs thro' B. Which was firſt to be proved.

OV being parallel to FG (*Def.* 17.) is in the ſame Plane of the Triangle OFG. Therefore *fg* continued will paſs thro' V. Which was alſo to be proved.

This Theorem being the principal Foundation of all the Practice of *Perſpective*, the Reader would do well to make it very familiar to him. To help him a little in his Reflections upon it, I have again repreſented the Senſe of it in *Fig.* 1. where the Projection *bc* meets the Original Line BC in its Interſection K, and paſſes alſo thro' its Vaniſhing Point V, which is produced by its Parallel OV.

N. B. When the Original Line it ſelf paſſes through its Vaniſhing Point, the whole Projection of it will be that Point; ſo that in that caſe the Line may be ſaid to vaniſh. This is one Reaſon for my uſing that Term. Another Reaſon is, that the further any Object is off, upon any Line, the ſmaller is its Projection, and at the ſame time, the nearer to this Point; and when it comes into this Point, its Magnitude vaniſhes, becauſe the Original Object is at an infinite Diſtance. This is eaſily conceived by imagining a Man to be going from you in a long Walk, who appears to be ſmaller and ſmaller, the further he goes. The Reaſon of this Diminution will appear from the following *Corollaries*.

COROL. 1. The Projections of all Original Lines that are parallel to one another, but not to the Picture, paſs thro' the ſame Vaniſhing Point: For they have but one Parallel common to them all, and conſequently but one Vaniſhing Point. This may be ſeen repreſented in *Fig.* 1. where the Projection *d a* and *c b* of the Parallel Lines D A and C B meet in their common Vaniſhing Point V.

COROL. 2. The Center of the Picture is the Vaniſhing Point of Lines perpendicular to the Picture. (See *Defin.* 7, 15, & 17.)

THEOR. IV.

The Projection of a Line parallel to the Picture, is parallel to its Original.

DEMONSTRATION.

Let the Plane E F (*Fig.* 5.) be the Picture, A B be the Original Line parallel to it, and *a b* its Projection; O being the Point of Sight, and O A B the Optic Cone. By *Theor.* 2. *a b* is the Interſection of the Picture with the Plane of the Triangle O A B. Therefore A B being parallel to the Plane E F, *a b* is parallel to A B. For they are both in the Plane of the Triangle O A B, and do not meet; for if they did, their common Interſection would be in the Plane E F, and conſequently A B would not be parallel to the Plane E F.

COROL. 1. The Projections of ſeveral Lines parallel to one another and to the Picture, are parallel to one another. Thus *a b* and *d c* are parallel to each other, and to their Originals A B and C D.

COROL. 2. The Projection *a b c d* of any Plane Figure A B C D parallel to the Picture, is ſimilar to its Original. For having drawn the Diagonal A C, and its correſponding Projection *a c*, the Sides *a b*, *b c*, *a c*, are parallel to their correſponding Originals A B, B C, A C; wherefore the Angles at *a*, *b*, and *c*, are equal to the correſponding Angles at A, B, and C; and conſequently the Triangle *a b c* is ſimilar to the Triangle A B C. For the ſame reaſon *a c d* is ſimilar to A C D; and conſequently the Figure *a b c d* is ſimilar to ABCD.

COROL. 3. In the ſame caſe, the Length of any Line *a b* in the Projection, is to the Length of its Original A B, as the Diſtance of the Picture is to the Diſtance between the Point of Sight and the Plane of the Original Figure. Let O *g* G be perpendicular to thoſe two Planes, cutting them in *g* and G. Then will *a b* : A B :: O *a* O A :: O *g* : O G (by *Prop.* 17. *Lib.* 11. *Elem.*) But *g* is the Center, O *g* is the Diſtance of the Picture, and O G is the Diſtance between the Point of Sight O, and the Original Plane A B C D. Wherefore *a b* is to A B, as the Diſtance of the Picture is to the Diſtance between the Point of Sight and the Plane of the Original Figure.

THEOR. V.

The Projection of a Line is parallel to its Director.

In *Fig.* 3. as already explain'd at the End of the Definitions, and in *Theor.* 3. the Lines O F, O G, O D, *f g*, are all in the ſame Plane. But the Directing Plane O D E is parallel to the Plane of the Picture A B I C (*Def.* 8.) Therefore the Director O D is parallel to the Projection *f g* (by *Prop.* 16. *Lib.* 11. *Elem.*)

COROL. 1. The Projections of Lines that have the ſame Director, are parallel to each other.

COROL. 2. When the Original Line is parallel to the Picture, its Director is parallel to it, and conſequently is in the Parallel of any Plane paſſing thro' that Original Line; and therefore the Vaniſhing Line of that Plane, and the Projection of the Line, are parallel to one another.

THEOR. VI.

The Vaniſhing Line, Interſection, and Directing Line of any Original Plane, are parallel to each other.

In the ſame *Fig.* underſtood as has been already explain'd, the Planes O V C and D F H being parallel (*Def.* 16.) as alſo O D E and C A B (*Def.* 8.) the Vaniſhing Line C V, the Interſection I B, and the Directing Line E D, are parallel to each other (by *Prop.* 16. *Lib.* 11. *Elem.*) Which was to be proved.

COROL. The Diſtance fV between the Projection of any Point, and the Vaniſhing Point V, is to the Diſtance B V, between the Interſection B, and the ſame Vaniſhing Point V, as the Diſtance O V of that Vaniſhing Point V, is to the Diſtance D F, between the Director and the Original Point F. For O V B D is a Parallelogram, wherefore B V is equal to D O, and the Triangles f O V and O F D are ſimilar, becauſe their Sides V f and O D are parallel. Wherefore fV : V O : : D O (= B V) : D F.

THEOR. VII.

The Vaniſhing Points of all Lines in any Original Plane, are in the Vaniſhing Line of that Plane.

DEMON-

DEMONSTRATION.

For ſince all the Original Lines are in the ſame Plane, their Parallels, which all paſs thro' the Point of Sight, will be all of them in the Parallel Plane (by *Prop.* 15. *Lib.* 11. *Elem.*) wherefore all the Vaniſhing Points are in the Vaniſhing Line.

COROL. 1. Original Planes, that are parallel, have the ſame Vaniſhing Line.

COROL. 2. The Vaniſhing Point of the common Interſection of two Original Planes, is the Interſection of their Vaniſhing Lines.

COROL. 3. The Vaniſhing Line of a Plane perpendicular to the Picture, paſſes through the Center of the Picture.

THEOR. VIII.

The Interſections of all Lines in the ſame Original Plane, are in the Interſection of that Plane.

This needs no *Demonſtration.*

COROL. 1. The Interſection of the common Interſection of two Original Planes, is the Interſection of their Interſections.

COROL. 2. Planes, whoſe common Interſection, is parallel to the Picture, have parallel Interſections, and alſo parallel Vaniſhing Lines.

PRO-

PROBLEM I.

Having given the Center and Diſtance of the Picture, to find the Projection of a Point whoſe Seat on the Picture, with its Diſtance from it, are given.

Let S (*Fig.* 6.) be the Center of the Picture, and *b* the given Seat of the Original Point. Draw at pleaſure S O equal to the Diſtance of the Picture, and parallel to it draw *b* A equal to the Diſtance of the Original Point from its Seat. Draw S *b* and A O meeting in *a*, which will be the Projection ſought.

DEMONSTRATION.

If the Angle O S *b* (and conſequently A *b a*) had been a Right Angle; then turning the Triangles S O *a* and *b* A *a* round the Line S *a b* as an Axis, till S O and *b* A become perpendicular to the Picture, O would be the Point of Sight, and A the Original Point, and A O would be the Viſual Ray cutting the Picture in *a*, which conſequently would be the Projection of the Point A, by *Theor.* 2. But the Point *a* is the ſame, whether the Angle O S *b* be a right Angle or not, becauſe the Triangles O S *a* and A *b a* are ſimilar, and therefore S *a* : *a b* : : S O : *b* A, which Proportionality is not affected by altering the Angle O S *b*. Therefore in all caſes the Point *a* thus found, is the Projection ſought of a Point, whoſe

Seat on the Picture is *b*, and its Diſtance from its Seat is *b* A.

COROL. 1. Having drawn S *b*, the Point *a* may be found by a Scale and Compaſſes dividing the Line S *b* in *a*, ſo that S *a* may be to *a b*, as the Diſtance of the Picture S O, is to the Diſtance *b* A of the Original Point from its Seat *b*.

COROL. 2. By this Propoſition the Projection of any Line may be found, by finding the Projections of two Points in it, and drawing a Line thro' thoſe two Projections.

PROBLEM. II.

To find the Projection of a Line, its Vaniſhing Point and Diſtance, having given its Seat, Interſection, and the Angle it makes with its Seat, and the Center and Diſtance of the Picture.

Let D E *Fig.* 6. be the given Seat of the Line propoſed, D its Interſection, and S the Center of the Picture. Draw D C making the Angle E D C equal to the Angle the Original Line makes with its Seat. Draw S V parallel to D E, and S O perpendicular to it, and equal to the Diſtance of the Picture. Then draw O V parallel to D C cutting S V in V, and draw D V. Then will V be the Vaniſhing Point and O V its Diſtance, and D V the indefinite Repreſentation of the Line propoſed.

DEMONSTRATION.

Imagine the Planes O S V and C E D to be turned round the Lines S V and D E as Axes, till they become perpendicular to the Picture. Then will O be the Point of Sight, and D C the Original Line; and O V being parallel to it, V is its Vaniſhing Point (by *Def.* 17.) and conſequently D V is its Projection (by *Theor.* 3.) Which was to be proved.

COROL. 1. D A C being conceived as the Original Line laid on the Picture, by turning the Plane C D E round the Line E D, the Projection of any part of it A C may be found by drawing the Lines A O and C O as Viſual Rays, cutting D V in *a* and *c*. For the Points *a* and *c* depend only on the Parallelism of the Lines O V and D C, and their proportion; *a* V being to *a* D as V O is to D A, and *c* V : *c* D : : V O : D C, upon account of the ſimilar Triangles *a* V O and *a* D A, and *c* V O and *c* D C. To underſtand this more clearly, the Reader may compare this Figure with *Fig.* 3. where the Points O, V, *f*, *g*, B, G, F, are analogous to the Points O, V, *c*, *a*, D, A, C, reſpectively.

COROL. 2. Having found D V, the Projection *c* of any Point C may be found by a Scale and Compaſſes, as making *c* V : *c* D : : O V : C D.

PRO-

PROBLEM III.

Having given the Projection of a Line, and its Vaniſhing Point; to find the Projection of the Point that divides the Original Line in any given Proportion.

Let A B (*Fig.* 7.) be the given Projection of the Line to be divided, and V its Vaniſhing Point. Draw at pleaſure V O and *b a* parallel to it, and thro' any Point O of the Line V O draw O A and O B cutting *b a* in *a* and *b*. Divide *a b* in *c* in the Proportion given, and draw O *c* cutting A B in C. Then will C be the Projection ſought, the Original of BC being to the Original of C A, as *b c* is to *c a*.

DEMONSTRATION.

O V being parallel to *b a*, *b a* may be conſider'd as the Original Line and O V as its Parallel, and conſequently O as the Point of Sight, and *a* O, *b* O, *c* O, as Viſual Rays projecting the Points A, B, C.

COROL. The Mathematical Reader will eaſily find, that C A × B V : C B × A V : : *c a* : *c b*. Whence the Point C may be found by a Scale and Compaſſes, making C A : C B : : A V × *c a* : B V × *c b*.

For $\frac{CA}{CB} = \left(\frac{CA}{ac} \times \frac{aC}{Cb} \times \frac{Cb}{CB} = \frac{AV}{VO} \times \frac{ac}{cb} \times \frac{OV}{VB} = \right) \frac{AV}{VB} \times \frac{ac}{cb}$. That is, C A : C B :: A V × *a c*. V B × *c b*.

PROBLEM IV.

Having given the Projection of a Line, and its Vaniſhing Point; from a given Point in that Projection, to cut off a Segment, that ſhall be the Projection of a given Part of the Original of the Projection given.

Let A B (*Fig.* 8.) be the Projection given, V its Vaniſhing Point, and C the Point from whence is to be cut off the Segment. Draw at pleaſure V O and *a b c* parallel to it, and from any Point O in V O draw O A, O B, O C cutting *a b* in *a*, *b*, *c*. Make *c d* to *a b*, as the given Part is to the Original of A B, and draw O *d* cutting A B in D. Then will C D be the Segment ſought.

DEMONSTRATION.

If O V be conceived as the Vaniſhing Line of any Plane paſſing through the Original of A B V; *a b c d* being parallel to it, may be conceived as the Projection of a Line parallel to the Picture (by *Cor.* 2. *Theor.* 5.) and therefore its Parts *a b* and *c d* will be in the ſame proportion to one another as their Originals (by *Theor.* 4.) But becauſe of the Vaniſhing Point V, the Originals of O *a*, O *b*, O *c*, O *d* are parallel (by *Cor.* 1. *Th.* 3.) Wherefore the Original of C D is to the Original of A B, as *c d* is to *a b* (by *Prop.* 2. *Lib.* 6. *Elem.*) Which was to be proved.

N. B. This Propoſition might have been demonſtrated as the foregoing, and the foregoing may be conſider'd as a particular Caſe of this, *viz.* when the Point C of this Propoſition coincides with one of the Points A or B.

COROL. The Point D may be found by a Scale and Compaſſes, making $DC : DV :: dc \times AB \times CV : ab \times AV \times BV$.

PROBLEM V.

Having given the Center and Diſtance of the Picture; to find the Vaniſhing Line (with its Center and Diſtance) of a Plane, whoſe Interſection is given, with the Angle of its Inclination to the Picture.

Let A B (*Fig.* 9.) be the given Interſection of the Plane, and C the Center of the Picture. Draw C O parallel to A B, and equal to the Diſtance of the Picture, and draw C A perpendicular to A B. Draw O S cutting A C in S, ſo that the Angle O S C may be equal to the Inclination of the original Plane to the Picture. Draw S D parallel to A B. Then will S D be the Vaniſhing Line ſought, S its Center, and O S its Diſtance.

DEMONSTRATION.

Imagine the Triangle O S C to be raiſed up on the Picture, ſo that O C may be perpendicular to the Picture. In that caſe O will be the Point of Sight, and S D being parallel to A B, a Plane paſſing thro' the Line S D and the Point O will be the Parallel of any original Plane paſſing thro'

A B, and inclined to the Picture in the Angle O S C. Wherefore S D is the Vaniſhing Line ſought. And O S, in that ſuppoſition, being perpendicular to S D, S is the Center, and S O the Diſtance of the Vaniſhing Line S D. Which was to be proved.

N. B. Taking O C for Radius, C S is the Cotangent, and O S the Co-ſecant of the Inclination of the Original Plane to the Picture.

PROBLEM VI.

Having given the Interſection of an Original Plane, with its Vaniſhing Line, its Center and Diſtance; to find the Projection of any Line in the Original Plane, having the Original Figures drawn out in their juſt Proportions.

Let D F (*Fig.* 10.) be the Interſection given, H G the Vaniſhing Line, and S its Center. Draw S O perpendicular to G H, and equal to the Diſtance of the Vaniſhing Line G H, and let the Space X be the Original Plane, ſeen on the Reverſe, as Objects appear in a Looking Glaſs; the Space Y being the Parallel Plane in the ſame manner folded down on the Picture; and let A B be the Original Line, whoſe Projection is ſought. Let A B cut the Interſection in D, and draw O G parallel to A B, cutting the Vaniſhing Line in G. Draw D G, which will be the indefinite Projection of A B. Thro' A and B draw at pleaſure A C and B C meeting in C, and in the ſame manner find their indefinite Projections F I, and E H cutting D G in *a* and *b*. Then will *a b* be the determinate Projection of A B, *a* being the Projection of the Extremity A, and *b* the Projection of the Extremity B.

Otherwiſe. Suppoſe K L to be the Original Line given. Having found its indefinite Projection Q G, as before, draw O K and O L cutting it in *k* and *l*, which will be the Projections of the Extremities K and L.

Otherwiſe by the Directors.

Let D F (*Fig.* 11.) be the Interſection given. And let the Original Plane be folded down on the Picture, ſo as to bring the Directing Line into the Plane H I, the Diſtance between A F and H I being equal to the Diſtance of the Vaniſhing Line given. Let alſo O be the Point of Sight brought into the Picture at the ſame time, along with the Directing Plane H O I. To find the indefinite Projection of any Original Line A B, continue it till it cuts E F in F and H I in G; then draw O G, and F *a* drawn parallel to it will be the indefinite Projection ſought. Then finding in the ſame manner the indefinite Projection E *d* of any other Line A D paſſing thro' A, by its Interſection with F *a* is got the Projection *a* of the Extremity A. And in the ſame manner is got the other Extremity *b*. Or thoſe Extremities might be found by drawing Lines from A and from B to O as in the foregoing Conſtruction.

DEMONSTRATION.

Imagine the Figures to be folded, *Fig.* 10. in D F and H G, and *Fig* 11. in E F and H I, till the Original Plane, its Parallel and the Directing Plane, and along with them the Point of Sight O, come into their proper Places. Then you will find, that D in *Fig*. 10 and F in *Fig*. 11. will be the Interſection of A B, and G in *Fig*. 11. will be its Directing Point. But O G in *Fig*. 10. is ſtill parallel to A B, wherefore G is its Vaniſhing Point, and D G its indefinite Projection (by *Theor*. 3.) and F *a* in *Fig*. 11. is ſtill parallel to O G, which is the Director of A B; wherefore F *a* is the indefinite Projection of A B (by *Theor*. 5.) That *a* found by the Interſection of F I with D G in *Fig*. 10. and of E *d* with F *a* in *Fig*. 11. is the Projection of the Interſection of the Original Lines A B and A C in *Fig*. 10. and of A B and A D in *Fig*. 11. is obvious. The other Conſtruction by the Lines A O is the ſame as by the Lines A O and C O for finding the Points *a* and *c* in *Fig*. 6. as is explain'd in *Cor*. 1. *Prob*. 2.

N. B. 1. The Reader that is not uſed to Mathematical Subjects may help himſelf in conceiving the manner here deſcribed of bringing the Original Plane, its Parallel, and the Directing Plane, with the Point of Sight, into the Plane of the Picture, by conceiving in *Fig*. 3. thoſe ſeveral Planes to be brought together by enlarging the Angles OVB and ODB, till the Planes lie flat on one another, and the Original Plane is ſeen on the back-ſide.

N. B. 2. In *Fig*. 11. the Projections *a d* and *k l* are parallel, their Originals having both of them the ſame Director O H, according to *Cor*. 1. *Theor*. 5. The ſame may be obſerved in *l m* and *c d*, which have the ſame Director O I.

PROBLEM VII.

Having given the ſame things as in the foregoing Problem, to find the Projection of any Figure in the Original Plane.

This is done by finding the Projections of the ſeveral Parts of the Figure given, by the foregoing Problem.

For example, the Projection *k l m n p* (*Fig.* 10.) of the Pentagon K L M N P is found thus. Drawing O G, O H, O I, O V parallel to K L, L M, M N, K P reſpectively, the Points G, H, I, and V, are their Vaniſhing Points. And K L, L M, M N, being continued cut the Interſections in their Interſections, Q, R, T. Whence drawing Q G, R H, T I, are got the Projections *l*, *m* of the Points L and M, by their mutual Interſections. Then drawing O K and O N, are got the Points *k* and *n*. Then drawing *k* V to the Vaniſhing Point V of K P, is got the indefinite Projection of K P. Laſtly drawing O P is got the Point *p*.

The Projections of Curvelined Figures are to be got by finding the Projections of ſeveral of their Points, and afterwards joining them neatly by hand. Thus in *Fig.* 13. *n.* 1. D E being the Interſection, and V F the Vaniſhing Line, and O the Point of Sight, and A B C an Original Circle, placed as in the foregoing Problem, the Projection *a* of any Point A may be found by drawing at pleaſure A D, and O V parallel to it; then drawing D V and O A meeting in the Point ſought *a*, according to the Conſtruction in *Problem* 6. D being the Interſection, and V the Vaniſhing Point of the Line AD. And the ſeveral Lines AD being drawn parallel to one another, the ſame Vaniſhing Point V may ſerve for them all.

Or as in *N.* 2. VF being the Directing Line brought into the Picture, as in *Fig.* 11. and the reſt remaining as before; drawing at pleaſure AD cutting DE and VF in D and V, then drawing OV and D parallel to it, the Projection *a* is got by drawing OA cutting D*a* in *a*. And the ſame Point V being uſed for all the Points A, all the Lines D*a* will be parallel to one another, and to the ſame Line OV.

PROBLEM VIII.

To find the Projection of any Figure in a Plane parallel to the Picture.

The Projection being ſimilar to its Original (by *Cor.* 2. *Theor.* 4.) this is done by making an exact Copy of the Original Figure; making the Homologous Sides in the Proportion explain'd in *Cor.* 3. of the ſame *Theorem.*

PROBLEM IX.

Having given the Interſection of a Plane, and its Vaniſhing Line, with its Center and Diſtance; to find the Original of any Projection given on the Picture.

Every thing being diſpoſed in *Fig.* 10. as in *Problem* 6, and 7. let it be propoſed to find the

Original of the Figure *k l m n p*. Having continued the Projections *k l*, *l m*, *m n* till they cut the Interſection and Vaniſhing Line in their Interſections Q, R, T, and their Vaniſhing Points G, H, I, and *k p* in its Vaniſhing Point V, draw O G, O H, O I, O V, and Q K, R M, T N parallel to the three firſt reſpectively meeting in L and M, which will be the Originals of *l* and *m*. Draw O *k* and O *n*, which will cut Q L and T M in the Originals K and N, of *k* and *n*. Then draw K P parallel to O V, and O *p* cutting it in P, which will be the Original of the Point *p*. Laſtly drawing N P, you have the Original Figure ſought K L M N P.

DEMONSTRATION.

This Conſtruction is evident from *Problem* 7. it being the Reverſe of it.

N. B. In the ſame manner one may go back to the Original Figure by the Directors, as in *Fig.* 11.

PROBLEM X.

The ſame things being given, to find only the Length of the Original of a Projection given.

Let I II (*Fig.* 10.) be the Projection given; the reſt of the Figure being to be underſtood as in the foregoing Problems. Continue I II till it cuts the Vaniſhing Line in its Vaniſhing Point V. And draw V O. In the Vaniſhing Line take V 3 equal to V O, and draw 3 I and 3 II cutting the Interſection in 1 and 2. Then will 1 2 be the Length ſought of the Original of I II.

DEMONSTRATION.

Let W be the Interſection of I II. V 3 being equal to V O the Diſtance of the Vaniſhing Point V, and W 2 being parallel to V 3, the Point 3 may be conſider'd as the Point of Sight, and W 1 . 2 as the Original Line and 3 . 1 and 3 . 2, as Viſual Rays producing the Projection I II.

COROL. The Length 1 . 2 may be found by a Scale and Compaſſes, making 1 . 2 : V 3 (or to V O) : : I II × W V : IV × IIV. For it is $\frac{1.2}{V3} = \frac{1.2}{d\,I} \times \frac{d\,I}{III} \times \frac{III}{V3} = \left(\frac{1.3}{I3}\right) \frac{WV}{IV} \times \frac{V3}{IIV} \times \frac{III}{V3} = \frac{WV}{IV} \times \frac{III}{IIV}$. That is, 1 . 2 : V 3 : : III × W V : IIIV × IV.

PROBLEM XI.

Having given the Vaniſhing Line of a Plane, its Center and Diſtance, and the Projection of a Line in that Plane; to find the Projection of another Line in that Plane, making a given Angle with the former.

Let O (*Fig.* 10.) be the Point of Sight placed as in the foregoing Problems, G H being the Vaniſhing Line, and *a b* the given Projection; it being required to draw *a c*, ſo that the Original of the Angle *b a c* may be equal to a given Angle. Continue *a b* to its Vaniſhing Point G. Draw G O, and O I making G O I equal to the given Angle, and cutting the Vaniſhing Line in I. Then draw *I a c*, which will be the Line ſought.

DEMON-

DEMONSTRATION.

The Figure being underſtood as in the foregoing Problems, let A B be the Original of *a b*, and conſequently be parallel to O G (by *Def.* 15. and *Theor.* 3.) For the ſame reaſon A C parallel to O I is the Original of *a c*, I being its Vaniſhing Point (by *Theor.* 3.) But A B and A C being parallel to O G and O I, the Angle BAC is equal to G O I, which is equal to the given Angle by the Conſtruction. Wherefore *b a c* repreſenting the Angle B A C, repreſents that given Angle. Which was to be done.

N. B. If it had been required to make *a b c* to repreſent the Angle A B C, the Angle G O H muſt have been made equal to the Complement of the Angle A B C to two Right Angles.

PROBLEM XII.

Having given the Vaniſhing Line of a Plane, its Center and Diſtance, and the Projection of one Side of a Triangle of a given Species in that Plane; to find the Projection of the whole Triangle.

The Projections of the Sides wanting are to be found by the foregoing Problem, the Angles of the Triangle being given. Thus having given the Projection *a b* (*Fig.* 10.) of the Side A B of the Triangle A B C, the Vaniſhing Point I of the Side *a c* is found by making the Angle I O G equal to the Angle C A B, and the Vaniſhing Point H of the Side *b c* is found by making the Angle H O G equal to the Complement of the Angle C B A to two Right Angles.

N. B. If the Vaniſhing Point of the Line given *ab* is out of reach, you may proceed thus. Taking any Line D R (parallel to the Vaniſhing Line H G by *Theor. 6.*) for the Interſection, by means of two Lines H *b* E, I *a* F, drawn at pleaſure thro' *b* and *a*, find the Originals A and B of the Points *a* and *b* (by *Prob. 9.*) and draw A B. Then on the Side A B compleat the Original Triangle, and find the Projections of the Sides wanting by *Prob. 7.*

PROBLEM XIII.

Having given the Vaniſhing Line of a Plane, its Center and Diſtance, and the Projection of one Side of any Figure in that Plane, to find the Projection of the Whole Figure.

Reſolve the whole Figure given into Triangles, by means of Diagonals, and find the Projections of thoſe Triangles one after another (by *Prob.* 12.) beginning with thoſe that have the Line given for one of their Sides.

The ſame thing may be done ſeveral Ways by the Application of the foregoing Problems, as is moſt convenient in every particular Caſe. This will be beſt underſtood by a few Examples.

Example I.

Fig. 14. In this Example I K is the Vaniſhing Line, S its Center, and S O its Diſtance, and A B parallel to IK is the given Projection of one ſide of a regular Hexagon. Having drawn OG parallel to IK (by *Def.* 15.) the Original of AB being parallel to the Picture (by *Cor.* 2. *Theor.* 5.) the Vaniſhing Points H, I, K of the Sides and Diagonals BC, FE, AD; AF, BE, CD; AC, are found by *Prob.* 11. making the Angles HOG 60 degr. IOG 120 degr. KOG 30 degr. Then drawing AK and BH, is got the Point C; drawing AH and CI is got D; drawing DE parallel to IK, and AS, is got E (for S is the Vaniſhing Point of AE, the Original of the Angle EAB being a Right Angle, as is GOS.) Laſtly drawing EH and AI is got F, which compleats the Figure ſought.

Example II.

Fig. 15. In this Figure the Projection *mrptqs* of the Figure MRPTQS (which is the Ichnography of a Regular Icoſaedron repoſing on one of its Faces,) is found, having given the Projection *ab* of the Side AB, VX being the Vaniſhing Line, and O the Point of Sight reduced to the Picture, as in the foregoing *Problems.* The Original Ichnography is deſcribed by making two concentric and parallel regular Hexagons, AFBICH, and RMSQTP, whoſe homologous Sides are in the Proportion of the Parts of a Line cut in extream and mean proportion, (See *Def.* 3. *Lib.* 6. *Elem.*) and drawing the Lines, as is obvious enough in the Figure.

Having continued *ab* to its Vaniſhing Point V, the Vaniſhing Points W and X of the other two Sides of the Triangle *abc* are found by

Prob. 11. Then drawing at pleaſure s p parallel to V X, and drawing W *a* and W *b* cutting it in a and b, and dividing a b in k, d, e, l, in the ſame proportion as AB is divided in K, D, E, L, and draw k W, d W, e W, l W, are got the Projections *k*, *d*, *e*, *l* of the Points K, D, E, L, (by *Prob.* 3. Then drawing *d* X, *e* W, and *e* X, are got the Points *f*, and *g*; and drawing *g* V, are got the Points *h* and *i*. Drawing *k* X and *l* W, is got the Point *m*, and *n* (which is the Projection of *N*) by the Interſection of *k* X with *d* W already drawn. Then making d o and o p each equal to m d, and drawing o W, p W, are got the Points *o* and *p* (by *Prob.* 3.) Then drawing *p* V is got the Point *q*, by its Interſection with *l* W already drawn. Drawing *o* W and *n* V is got *r*: And making m s equal to m d, and drawing s W is got *s*. Laſtly, drawing *s* X cutting *r o* W already drawn, is got *t*. The reſt is done by joining the Points found, as is evident enough in the Scheme.

Example III.

Fig. 16. In this Example is found the Projection of the Ichnography of a regular Dodecaedron, having the Projection of one Side given, by returning to the Original Figure, by *Prob.* 9. and then proceeding by *Prob.* 7. I ſhall leave the Reader to exerciſe himſelf in conſidering this Scheme, and only obſerve, that the Original Ichnography is made by deſcribing two concentric and parallel Decagons, whoſe homologous Sides are as the Segments of a Line cut in extream and mean Proportion.

Example IV.

Fig. 17. In this Figure D C being the Vaniſhing Line, and O the Point of Sight reduced to the Picture, as in the foregoing Problems, the Projection A N B M L P, of a regular Octaedron, having given the Projection of one of the Sides A B, is found as follows. Having continued A B to its Vaniſhing Point C, the Vaniſhing Point G of the Sides A K and N M; found by *Prob.* 11. making the Angle C O C equal to 60 degr. Then (by *Prob.* 10.) taking any Line *b l* (parallel to C D) for the Interſection, and making C D equal to C O, and G H equal to G O, and drawing D A and D B cutting *b l* in *a* and *b*, *a b*, is the Length of the Original of A B, and drawing H A cutting *b l* in a, and making a *l* equal to *a b*, and drawing *l* H cutting A G in L, is got the Projection A L (the Original of it being equal to a *l*, and conſequently to the Original of A B, a *l* and *a b* being equal. Then dividing *a b* and a *l* each into three equal Parts by the Points *e*, *f*, *i*, *k*, and drawing *e* D, *f* D, *i* H, *k* H, are got the Points E, F, I, K (by *Prob.* 3.) Then drawing F G, K H, E I, are got the Points M, N, P, which compleat the Figure.

PROBLEM XIV.

Having given the Center and Diſtance of the Picture, and the Vaniſhing Line of a Plane; to find the Vaniſhing Point of Lines perpendicular to that Plane.

Let A B (*Fig.* 18.) be the Vaniſhing Line given, and C the Center of the Picture. Draw C A perpendicular to A B, and C O parallel to it, and equal to the Diſtance of the Picture. Draw AO and O D perpendicular to it, cutting C A in D; which will be the Vaniſhing Point ſought.

DEMONSTRATION.

Imagine the Triangle A O D to be turned up on the Plane of the Scheme, ſo that C O may be perpendicular to it, O being brought into the Point of Sight. This being done, the Plane paſſing through the Point O and the Line A B will be the Parallel of the Original Plane, and the Line O D will be perpendicular to it; and conſequently will be the Parallel of Lines perpendicular to that Original Plane. Wherefore D is the Vaniſhing Point of thoſe Perpendiculars (by *Def.* 17.)

N. B. 1. When the Vaniſhing Line A B paſſes through the Center of the Picture, that is, when the Original Plane is perpendicular to the Picture, the Point D will be infinitely diſtant, the Line O D being parallel to A D, and the Projections of the Lines perpendicular to the Plane propoſed will all of them be perpendicular to A B, they being to meet the Line A C, which is perpendicular to it at an infinite diſtance; and conſequently they will be parallel to one another. Which they ought to be upon another account, their Originals being all parallel to the Picture.

N. B. 2. But when the Original Plane is parallel to the Picture, the diſtance C A will be infinite, and conſequently O A will be parallel to

C A, and O D will coincide with O C, making the Point D to fall into the Center of the Picture C, agreeably to *Cor.* 2. *Theor.* 3.

N. B. 3. C D is a third proportional to A C and C O; as alſo is A D to A C and A O.

N. B. 4. O D is the Diſtance of the Vaniſhing Point D.

PROBLEM XV.

Having given the Center and Diſtance of the Picture, to find the Vaniſhing Line, its Center and Diſtance, of Planes that are perpendicular to thoſe Lines that have a certain given Vaniſhing Point.

Let C (*Fig.* 18.) be the Center of the Picture, and D the Vaniſhing Point given. Draw D C, and C O perpendicular to it, and equal to the Diſtance of the Picture. Then draw D O, and O A perpendicular to it, cutting D C in A. Perpendicular to D C draw A B, which will be the Vaniſhing Line ſought, A being its Center (by *Theor.* 1.) and O A its Diſtance.

This Conſtruction follows neceſſarily from the Conſtruction of the foregoing Problem, and the Obſervations at the End of that may be applied here.

PROBLEM XVI.

Having given the Center and Diſtance of the Picture, through a given Point to draw the Vaniſhing Line of a Plane that is perpendicular to another Plane whoſe Vaniſhing Line is given, and to find the Center and Diſtance of that Vaniſhing Line.

Let A B (*Fig.* 18.) be the Vaniſhing Line given, and C the Center of the Picture, and let E be the Point given. Find the Vaniſhing Point D of Lines perpendicular to the Original Planes of A B, (by *Prob.* 14.) Draw D E, which will be the Vaniſhing Line ſought. Draw C F cutting D E at Right Angles in F, and F will be the Center of the Vaniſhing Line D E (by *Theor.* 1.) Make a Right-angled Triangle, whoſe Baſe is C F, and its Perpendicular is equal to the Diſtance of the Picture, and its Hypothenuſe will be the Diſtance of the Vaniſhing Line D E (by *Cor. Th.* 1.)

DEMONSTRATION.

Becauſe the Plane, whoſe Vaniſhing Line is ſought, is perpendicular to the other Plane, its Vaniſhing Line muſt paſs thro' the Vaniſhing Point D of Lines perpendicular to that other Plane, becauſe ſome of thoſe Lines are in the Plane ſought. Therefore D E is the Vaniſhing Line ſought. The reſt needs no Demonſtration.

COROL. 1. If F C be continued till it cuts the Vaniſhing Line given in B, B will be the Vaniſhing Point of Lines perpendicular to the Original Plane of the Vaniſhing Line D E. For that Vaniſhing Point is in the Line F C by the Conſtruction of *Problem* 14. and it is in the Vaniſhing Line given by the Demonſtration of the preſent Problem.

COROL, 2. And therefore if the Vaniſhing Lines A B and D E meet in G, the Points B, D, and G will be the Vaniſhing Points of the three Legs of the ſolid Angle of a Cube, which are perpendicular to one another. And drawing D B; B G, G D, and D B will be the Vaniſhing Lines of the three Planes that contain that ſolid Angle.

COROL. 3. The Diſtance of the Vaniſhing Line D G is equal to the Line F P, the Point P being the Interſection of the Line F C with a Circle deſcribed on the Diameter D G.

PROBLEM XVII.

Having given the Center and Diſtance of the Picture, and the Vaniſhing Point of the common Interſection of two Planes that are inclined to one another in a given Angle, and the Vaniſhing Line of one of them; to find the Vaniſhing Line of the other of them.

Let C (*Fig.* 18.) be the Center of the Picture, B G the given Vaniſhing Line of one of the Planes, and B the Vaniſhing Point of their common Interſection, and H the Angle of their Inclination to one another. Find the Vaniſhing

C D

Line GD, of Planes perpendicular to the Lines whoſe Vaniſhing Point is B (by *Prob.* 15.) Let that Vaniſhing Line cut the Vaniſhing Line given in G. In GD find the Vaniſhing Point E of Lines making the given Angle H with the Lines whoſe Vaniſhing Point is G (by *Prob.* 11.) that is, in BCF perpendicular GFD take FP equal to the Diſtance of the Vaniſhing Line GD (found by *Prob.* 15.) and draw PG, and PE making the Angle EPG equal to H. Draw BE, which will be the Vaniſhing Line ſought.

DEMONSTRATION.

Imagine the Triangle GPE to be turned up on the Line GE, ſo that the Point P may be in the Point of Sight perpendicular over the Center of the Picture C. In that caſe the Planes BPG, GPD, DPB will be the Parallels of three Original Planes, whoſe Vaniſhing Lines are BG, GD, DB; that whoſe Vaniſhing Line is DG being perpendicular to the other two (by the Conſtruction, becauſe it is perpendicular to their common Interſection, whoſe Vaniſhing Point is B.) Therefore the Original Planes, whoſe Vaniſhing Lines are BG and BD, are inclined to one another in the Angle EPG, that is, in the Angle H, (for the Inclination of two Planes is always meaſured in a Plane perpendicular to their common Interſection.) Therefore BG being the Vaniſhing Line given, BE is the Vaniſhing Line ſought.

N. B. The Center of the Vaniſhing Line B E is found by drawing a Line perpendicular to it from C (by *Theor.* 1.) and then its Diſtance is found, as was found the Diſtance P F in *Prob.* 16.

PROBLEM XVIII.

Having given the Center and Diſtance of the Picture, and the Vaniſhing Line of one Face of any ſolid Figure propoſed, and the Projection of one Line in that Face; to find the Projection of the whole Figure.

By means of the Projection given find the Projection of the Ichnography of the Figure propoſed on the Plane of that Face, whoſe Vaniſhing Line is given (by *Prob.* 13.) Then (by *Prob.* 16.) find the Vaniſhing Line of the Plane of the Orthography and deſcribe the Projection of the Orthography by the help of the Lines already given in the Ichnography (by *Prob.* 13.) Laſtly, by the Interſections of the Projections of Perpendiculars to the Ichnography and Orthography, will be found the ſeveral Points of the Projection required.

Otherwiſe. Having found the Projection of the Face whoſe Vaniſhing Line is given, by means of the Projection of the Line given, find the Vaniſhing Lines of the adjacent Faces (by *Prob.* 17.) and deſcribe their Projections by the help of the Lines given in the Projection of the firſt Face; and ſo on, till the whole Projection ſought is compleated.

This is a general Deſcription of the Method to be uſed in putting any Figures propoſed into *Perſpective*; but in the Practice of particular Caſes ſeveral Expedients may be uſed, in the various Application of the foregoing Problems. But all this will be beſt underſtood by Examples.

Example V.

Fig. 19. In this Figure is found the Projection of a regular Dodecaedron, having given the Projection A B of one Side parallel to the Picture, by means of the Ichnography and Orthography; F G (parallel to A B) being the given Vaniſhing Line of the Face A B C D E, F its Center, H the Center of the Picture, and H O its Diſtance. To avoid Confuſion of Lines, the Ichnography and Orthography are removed from the Space taken up by the Projection ſought in the following manner. For the Ichnography; drawing at pleaſure *a b* parallel to A B, and at a ſufficient diſtance from it, and then drawing I A and I B cutting *a b* in *a* and *b*, the Line A B is transfer'd to *a b*; I being the Vaniſhing Point of Lines perpendicular to the Face A B C D E, whoſe Vaniſhing Line is F G (found by *Prob.* 14.) This being done, the whole Ichnography is deſcribed on the Line *a b*, by *Prob.* 13. as it is deſcribed in *Ex.* 3.

For the Orthography; I H F paſſing thro' the Center of the Picture, is taken for its Vaniſhing Line, as being moſt convenient; the Orthography in this caſe being the moſt ſimple, and the Projections of Lines perpendicular to it being all of them perpendicular to F I (by *Note* 1. *Prob.* 14.) Having drawn G A (from the Vaniſhing

Point G of the Line *a e* in the Ichnography) and *e* I cutting it in E, is got the Projection A E. Then drawing A a and E e both parallel to G F (and conſequently repreſenting Perpendiculars to the Orthography, as is already ſaid) and a e at pleaſure paſſing thro' F, and cutting them in a and e, is got a e; by the means of which the whole Projection of the Orthography is deſcribed (by *Prob.* 13.)

Having thus got the Projections of the Ichnography and Orthography, any Point K of the Projection ſought is got by drawing k K parallel to F G and *k* I, from the correſponding Points k and *k*, meeting each other in K.

For underſtanding the Projection of a Cube, which appears in this Figure, after what has been already ſaid, it is ſufficient to inform the Reader, that two of its Vaniſhing Lines are F G, and F I, and the third is a Line paſſing thro' I parallel to F G.

As to the Shadows, which are ſuppoſed to be caſt by the Sun on the Plane of the Face A B C D E of the Dodecaedron; the Shadow *u v* of any Line V *v* is found as follows. S is the given Vaniſhing Point of all the Rays of Light, which being ſuppoſed to come from the Sun, are to be conſider'd as parallel. Therefore I S paſſing thro' the Vaniſhing Point I of the Line V *v* and the Vaniſhing Point S of the Rays, is the Vaniſhing Line of the Plane made by all the Rays paſſing thro' the Line V *v*, and projecting the Shadow *v u*. And I S cutting the Vaniſhing Line F G of the Plane the Shadow is caſt on in *s*, *s* is the Vaniſhing Point of the Shadow *v u*, which is the

Interſection of the Plane of the Shade (whoſe Vaniſhing Line is I *s*) and the Plane the Shadow is caſt on (*s* being the Vaniſhing Point of that Interſection by *Cor.* 2. *Th.* 7.) Having therefore drawn *v s*, and V S cutting it in *u*, *v u* is the Shadow of the Line *v* V.

N. B. In the Original of the Orthography in the preſent Figure, the Points e, m, s, p are the Angles of a Square, The Lines o n, a l, q r being parallel to e m, and o q, n r being parallel to e p, and equal to a l, t k, o n, q r, are equal, and o n, e m, a l, are in the continued Geometrical Proportion, of the leſſer Segment to the greater, of a Line cut in extream and mean Proportion.

Several Lines mention'd in this Scheme are not actually drawn, to avoid Confuſion.

Example VI.

Fig. 20. In this Scheme the Vaniſhing Line of the Ground, which the Piece of Building, *&c.* ſtand upon, is A I C K B paſſing through the Center of the Picture C, the Diſtance of the Picture being equal to C O. B G and A H are the Vaniſhing Lines of the upright Planes a b d e, D E N, *&c.* and *a b c*, D F *n m*, *&c.* G and H being the Vaniſhing Points of the Lines *b e*, and *m n*, which touch the upper Corners of the two Flights of Steps, and conſequently A G and B H are the Vaniſhing Lines of the Planes that touch the upper or the under Edges of the Steps.

The given Side *h r* of the Baſe of the regular Tetraedron is parallel to the Vaniſhing Line A B *u*, which is the Projection of the Center of the Baſe *h r p*, and the Seat of the Vertex *o*, is found by drawing *p q* parallel to A B, and I *r* cutting it in *q*, and then drawing *C p* and *h q* meeting in *u*. C L perpendicular to A B is the Vaniſhing Line of a Plane perpendicular to the Baſe *k r p* ſtanding on *u p*, and paſſing thro' the Line *p o*, whoſe Vaniſhing Point is L, found (by *Prob.* 11.) by making C Q equal to the Diſtance of the Picture, and the Angle L Q C equal to the Original of the Angle *u p o*. V X is the Vaniſhing Line of the Face *u p r*, by the help of which that Face is deſcribed, having given *p r* (by *Prob.* 12.) as the Face *h r p* was deſcribed on *h r*. The regular Octaedron, and Icoſaedron, are deſcribed by their Ichnography and Orthography, which Method is ſufficiently explain'd in the foregoing Example. I will only inform the Reader, that in the Orthography A B C D E F G H of the Icoſaedron, the Lines C D, A F, G H are equal, as alſo are A C, F D, B E, and A F is to B E, as the leſſer Segment is to the greater of a Line cut in extream and mean Proportion.

The Light is ſuppoſed to come from the Sun, and the Rays are parallel to the Picture and to the Line A M; ſo that the Shadow P of any Point D, is found by drawing N P through its Seat parallel to A B, and D P parallel to A M cutting it in L. For the Shadow of the Tetraedron, which is turned up on the End of the Steps; having in the ſame manner found *s*, (which would be the Shadow of *o* on the Ground, if the Steps were away) drawing *s h* and *s p*, are got the Shadows *h o* and *p o*. Let *s u* and *s p* cut the lower Edge of the Steps in *y* and *x*; then drawing *x t* perpendicular to A C and cutting *o s* in *t*, is got the Shadow

y t

Shadow *t* of the Vertex *o* on the End of the Steps, and drawing *x t* is got that part of the Shadow of *o p*, which falls on that End. And in the ſame manner is got the Shadow of *o h*. All the Rays being parallel to A M, and A being the Vaniſhing Point of D F, A M is the Vaniſhing Line of the Plane made by the Rays which paſs thro' the Line D F, and B M being the Vaniſhing Line of the Wall, the Shadow F *f* is caſt on; M is the Vaniſhing Point of the common Interſection of thoſe two Planes, that is, of the Shadow F *f* of the Line D F.

Example VII.

Fig. 21. In this Scheme A C B paſſing through the Center of the Picture, C is the Vaniſhing Line of the Ground; A is the Vaniſhing Point of the Edge E F which the Beam reſts upon, and the other Edges parallel to it; D is the Vaniſhing Point of the Edge G H, *&c.* and D B perpendicular to A B, is the Vaniſhing Line of the Plane E G H; P and Q are the Vaniſhing Points of the Sides M K and K N, of the regular Tetraedron. and conſequently P Q is the Vaniſhing Line of the Triangle K N M; L is the Light and A its Seat on the Ground. Having B E *g*, that is the Interſection of the upright Plane E G H with the Ground, and conſequently *g*, where H G meets it, is the Interſection of the Edge H G with the Ground. B E cuts the Edge S R of the Wall S R h in R, and conſequently R *h* perpendicular to A B is the Interſection of the Wall with the Plane E G H, and conſequently *h*, where R *h* and G H meet, is the Interſection of the Line

GH with the Wall. DL *l* is the Projection of a Line parallel to the Original of GH (becauſe of the Vaniſhing Point D) and B l is its Seat, wherefore *l* is its Interſection with the Ground.

Now the Originals of L *l* and HG being parallel, they are in the ſame Plane, *viz.* in the Plane which makes the Shadow of HG; but *lg* repreſents the Interſection of that Plane with the Ground; wherefore *lg* continued is part of the Shadow, and drawing LG cutting it in g, g is the Shadow of G, and g S is that part of the Shadow of HG which is on the Ground. Then drawing S*h* and LH cutting it in h, h S is the other part of that Shadow againſt the Wall.

Having drawn *gp* and DT both parallel to AB, D*g* and *gp* are the Projections of two Lines in a Plane, whoſe Vaniſhing Line is DT; and T is the Vaniſhing Point of the common Interſection of that Plane with the Plane of the Triangle KMN (by *Cor.* 2. *Th.* 7) PQT being its Vaniſhing Line. Therefore drawing T p cutting gD in V, V is the Interſection of the Line *g* GHD with the Plane of that Triangle KMN. Then *lg* cutting MK in *r*, drawing *rt* V, *rt* is that part of the Shadow of GH, which falls on the Triangle KMN.

Having thus explain'd the manner of finding the Shadow of GH, the reſt needs no Explanation.

Example VIII.

Fig. 22. In this Scheme C is the Center of the Picture, and CA the Vaniſhing Line of the Ground, and of the Surface of the Water which gives the Reflexions; and S is the Vaniſhing Point of the Rays of Light, which are ſuppoſed to come from the Sun.

The Shadow of the perpendicular Line B D is found thus: S A drawn perpendicular to C A gives the Vaniſhing Point A of the Shadow on the Ground B *f*. Then in the Circumference of the Baſe of the Cylinder (which is parallel to the Picture, its Axis being perpendicular to it, and conſequently having the Vaniſhing Point C) taking any Point E, and finding its Seat on the Ground *e*, drawing C E, and C *e* cutting B A in *f*, and then drawing *f* P perpendicular to C A and cutting C E in P, is got one Point P of the Shadow on the Surface of the Cylinder. And in the ſame manner are got all the other Points of that Shadow. To prove this, the Reader need only conſider, that the Original of *e* E P *f e* is an upright Plane cutting the Cylinder in E P, and *f* P is the Shadow of B D on that Plane. Wherefore P is the Point where that Shadow falls on the Surface of the Cylinder. Any Point Q of the Shadow of the Circumference of the inward Cylinder on its Surface, is found thus. Having drawn C S, parallel to it is drawn at pleaſure G H cutting that Circumference in G and H. Then drawing G S and C H meeting in Q, Q is the Point ſought. For C being the Vaniſhing Point of the Axis of the Cylinder, as well as of C H, H Q is in the Surface of the Cylinder, and G H being parallel to C S, is the Projection of a Line parallel to the Picture, in a Plane whoſe Vaniſhing Line is C S (and its Original being parallel to the Picture, is in the Baſe of the Cylinder, which is parallel to the Picture.) Therefore the Originals of H G, H C, and G S being in the ſame Plane, Q is the Projection of the Point where the Ray of Light, whoſe Projection is G S, cuts the Surface of the Cylinder; that is, it is the Projection of the Shadow of the Original of the Point G in the Circumference of the Baſe, on the inward Surface of the Cylinder.

b being the Seat of the Point D on the Surface of the Water, the Reflexion *d* of the Point D is found by continuing the Perpendicular D *b* till *b d* is equal to *b* D. This is evident, becauſe the known Law of Reflexions, is, that the Reflexions of all Objects appear to be as much on one Side of the reflecting Plane, as the real Objects are on the other Side of it. In A S, making A *s* equal to A S, any Point *q* in the Shadow on the Surface of the inward Cylinder in the Reflexion, is found in the ſame manner as Q in the real Figure, uſing the Point *s* inſtead of S.

The Shadow of the Cylinder on the Surface of the Cone, is found by ſuch another Expedient, as the Shadow of the Line B D on the Surface of the Cylinder.

Example IX.

Fig. 23. In this Scheme every thing elſe being eaſily to be underſtood by what has been already explain'd, I ſhall only ſhew the manner how the Reflection is found in the Looking Glaſs of the Picture on the Eazle.

A is the Center of the Picture, and A B the Vaniſhing Line of the Ground; the Diſtance of the Picture being equal to A B. A C is the Vaniſhing Line of the Picture on the Eazle, and CD the Vaniſhing Line of the Looking Glaſs.

Through *a*, where the Edge *b a* of the Leg of the Table cuts the Surface of it, drawing *a e*, and through *b* drawing *b d*, both parallel to A B, *b d* cutting the Interſection *c d* of the Surface of the Picture on the Eazle with the Ground in *d*, and then drawing *d e* parallel to A C, and cutting *a e* in *e*, and then drawing A *e*, is got the Projection A *e* of the common Interſection of the Surface of the Table, and of the Picture on the Eazle. For *a e* being parallel to A B, is the Projection of a Line in the Surface of the Table parallel to the Picture, and for the ſame reaſon *b d* is the Projection of a Line on the Ground, and *d e* is the Projection of a Line in the Plane of the Picture on the Eazle, both of them parallel to the Picture; *a b* is alſo the Projection of a Line parallel to the Picture. Therefore *a b d e* is the Projection of a Trapezium parallel to the Picture, whoſe Angle *e* is in the common Interſection of the Surface on the Table, and of the Picture on the Eazle. But A being the common Interſection of the Vaniſhing Lines of thoſe two Planes, is the Vaniſhing Point of their common Interſection, and therefore *e* A is the Projection of that Interſection (by *Cor.* 2. *Th.* 7.) For the ſame reaſon *o* being the Projection of the Point where the Surface of the Glaſs touches the Table, and E being the common Interſection of the Vaniſhing Lines A B and C D, *o* E is the Projection of the common Interſection of the Surface of the Table and the Surface of the Glaſs. Therefore *f* where *o* E and *e* A meet, is the Projection of the Point where the three Planes meet, of the Surface of the Table, the Glaſs, and the Picture on the Eazle. Therefore drawing *f* C, it is the Projection of the common Interſection of the Picture on the Eazle and the Looking Glaſs.

Having found the Vaniſhing Point P of Lines perpendicular to the Plane of the Looking Glaſs, whoſe Vaniſhing Line is C D (by *Prob.* 14.) drawing P A thro' the Vaniſhing Point A of the Line G H, and cutting C D in D, D is the Vaniſhing Point of the Seat of G H on the Plane of the Glaſs. Therefore G H cutting C *f* in *i*, D *i* is the Projection of that Seat. Then drawing G P cutting D *i* in *k*, *k* is the Seat of the Point G on the Glaſs. Wherefore in G P making *k g* to repreſent a Line equal to that repreſented by G *k* (by *Prob.* 3.) *g* is the Projection of the Reflection of G, and *g i* is the Reflexion of G *i*, and drawing P H cutting *g i* in *h*, *g h* is the Reflexion of G H. And in the ſame manner may be found any other Lines in the Reflexion.

The Reflexion of the Picture on the Eazle may alſo be deſcribed by its Vaniſhing Line, in the ſame manner as the Projection of the Picture itſelf was deſcribed; for in P A D making a D to repreſent a Line equal to that repreſented by A D, a is the Vaniſhing Point of the reflected Line *g h*, and C a is the Vaniſhing Line of the reflected Picture on the Eazle.

LINEAR PERSPECTIVE.

PART II.

Of the Manner of finding the Original Figures from their Projections given, and of the Situation that is neceſſary to be obſerved for viewing particular Projections.

PROBLEM XVIII.

Having given the Projection of a Line divided, and its Vaniſhing Point; to find the Proportion of the Parts of the Original.

LET AB (*Fig.* 7.) be the given Projection, divided in C, and V its Vaniſhing Point. Draw at pleaſure VO, and *a b* parallel to it, and from any Point O in the Line O V draw O A, O B, O C, cutting *a b* in *a*, *b*, and *c*. Then will the

Original of AC be to the Original of CB, as *ac* is to *cb*.

COROL. $ac:b::AC\times BV:BC\times AV$.

PROBLEM XIX.

Having given the Projection of a Line divided into two Parts, and the Proportion of the Original; to find its Vaniſhing Point.

Let AB (*Fig.* 17.) be the Projection given divided in C. Through C draw at pleaſure a C b, and in it make a C to C b, as the Original of A C is to the Original of C B, and draw a A and b B meeting in O. Parallel to a b draw O V cutting A B in V, which will be the Vaniſhing Point ſought.

Fig 17

COROL. $BV:BA::Ca\times CB:Cb\times AC - Ca\times CB$.

Theſe two laſt Problems, with their Corollaries, follow eaſily from *Prob.* 3. and its *Cor.*

PROBLEM XX.

Having given the Projection of a Triangle, with its Vaniſhing Line, its Center and Diſtance; to find the Species of the Original Triangle.

Let *a b c* (*Fig.* 10.) be the Projection given, H G its Vaniſhing Line, and S its Center, and S O perpendicular to H G, and equal to its Diſtance. Having continued the Sides of the Projection given, till they cut the Vaniſhing Line in their Vaniſhing Points G, H, I, draw G O, G H, and G I, and the Originals of the Angles *b a c*, *a b* H, *a c b*, will be equal to G O I, I O H, G O H, reſpectively (by *Prob.* 11.) Whence the Species of the Original Triangle is given.

PROBLEM XXI.

Having given the Projection of a Triangle of a given Species, and its Vaniſhing Line; to find the Center and Diſtance of that Vaniſhing Line.

Let A B C (*Fig.* 12.) be the given Projection, and F D its Vaniſhing Line. Continue the Sides of the Projection till they cut the Vaniſhing Line in their Vaniſhing Points D, E, F. Biſect D E and E F in G and H, and draw G I and H K perpendicular to F D, making G I to G E as Radius is to the Tangent of the Angle repreſented by B A C, and K H to E H as Radius is to the Tangent of the Angle repreſented by B C A; ſo that E I G and F K H may be equal to thoſe Angles. With the Centers I and K and the Radius's I E and K E, deſcribe two Circles cutting each other in O, and draw O S cutting F D at Right Angles in S. Then will S be the Center, and S O the Diſtance ſought.

DEMONSTRATION.

Suppoſing S to be the Center and S O the Diſtance of the Vaniſhing Line F D, the Originals of the Angles B A C and B C A will be equal to D O E, and E O F (by *Prob.* 11.) But by the nature of the Circle DOE and FOE are equal to GIE and HKE, which by the Conſtruction are equal to the Angles that ought to be repreſented by BAC and BCA. Therefore S is the Center and SO the Diſtance ſought.

PROBLEM XXII.

Having given the Projection of a Trapezium of a given Species; to find its Vaniſhing Line, Center and Diſtance.

Let *abcd* (*Fig.* 12.) be the Projection given Draw the Diagonals *a c*, *b d*, meeting in *e*, and by the Proportions of the Originals of *a e*, *e c*, and *b e*, *e d*, finding the Vaniſhing Points E and F, of the Lines *a c* and *b d* (by *Prob.* 19.) Draw FE, which will be the Vaniſhing Line ſought. Then by the given Species of the Original of the Triangle *a b e*, find the Center S and Diſtance SO (by *Prob.* 21.)

PROBLEM XXIII.

Having given the Projection of a Right-angled Parallelopiped; to find the Center and Diſtance of the Picture, and the Species of the Original Figure.

Let ABCDEFG (*Fig*, 24.) be the Projection given. Continue the Projections of the parallel Sides, till they meet in their Vaniſhing Points H, I, K, and draw HI, HK, IK, which will be the Vaniſhing Lines of the ſeveral Faces of the Figure ſought, containing a ſolid Right Angle. Draw KL perpendicular to HI, and HM perpendicular to KI meeting in S; which will be the Center of the Picture (by *Cor.* 2. *Prob.* 16.) Then on the Diameter LK deſcribe a Circle, and draw SO perpendicular to LK cutting it in O, and OS will be the Diſtance of the Picture (by *Prob.* 14. LOK being a Right Angle upon account of the Circle.) Laſtly, find the Diſtances of the Vaniſhing Lines KI and IH (by *Cor.* 3. *Prob.* 16. M and L being their Centers, by *Th.* 1.) and then find the Species of the Originals of the Faces DAFE and DABC (by *Prob.* 20.)

COROL. When the Vaniſhing Line of one of the Faces (ſuppoſe IH) paſſes through the Center of the Picture, the Vaniſhing Point K of the Sides perpendicular to it, will be at an infinite diſtance: by which means the Situation of LK will be indetermined. So that the Species of that Face ABCD may be taken at pleaſure, and then the Center and Diſtance of the Picture may be found by *Prob.* 20. And in this caſe, if it were only required that the Projection propoſed ſhould repreſent a right angled Parallelopiped in general, the Place of the Point of Sight might be any where in the Circumference of a Circle deſcribed on the Diameter HI, and in a Plane perpendicular to the Picture. This I leave as a hint that may be uſeful to the Painters of Scenes in Theatres.

FINIS.

APPENDIX.

Numb. I.

The Deſcription of a Method, by which the Repreſentations of Figures may be drawn on any Surface, be it never ſo irregular.

FROM what has been ſaid in this Book to explain the Principles of Painting, eſpecially at the End of the Definitions, it is evident that the Senſe of the ſecond Theorem may be extended to any Surface that Figures are painted on, be it Concave or Convex, or never ſo irregular. So that be the Surface of the Picture A B C (*Fig.* 3.) of any Form whatſoever, the Projection *f g* of the Original Line F G, will ſtill be the Interſection of the Picture with the Plane of the Triangle F G O. But the Parallel O V is in that Plane (*Th.* 3.) Hence it follows, that if the Flame of a Lamp be ſo placed, as to caſt the Shadow of O V on any Point B of the Line F G, it will cover the whole Line F G, and at the ſame time cover the Line V *f g* B on the

Picture, which is the Projection of F G; all the Rays coming from the Lamp, and paſſing by the Line O V, in that caſe making a Plane, which coincides with the Plane of the Triangle O F G. The ſame thing will happen, if a Perſon ſhould place his Eye ſo as to make any Point of the Line F G to ſeem to be cover'd by the Line O V; in that caſe O V will ſeem to cover all the Projection V *f g*. So that if a Lamp be ſo placed, as to make the Shadow of O V to paſs thro' any one Point of the Projection V *g*, it will coincide with the whole; and if a Perſon places himſelf ſo as to make O V to ſeem to cover any one Point of the ſame Projection, it will ſeem to cover the whole. Hence I imagine, that the following Method may be of uſe for drawing the Projections of any Figures on any Surface, ſuppoſe on the Walls and Cupola's of Churches; the Walls and Cielings of great Rooms, the Scenes of Theatres, *&c.*

Chuſe ſome Principal Line in the Deſign to be drawn, and having by ſome proper Method found the Projections of its extream Points, through the Point of Sight paſs a Thread parallel to the Original Line, and caſt the Shadow of it on thoſe two Points already found, and that Shadow mark'd with a Crion will be the Projection of that principal Line; or, if in any particular caſe it happens to be more convenient, place your Eye ſo as to make that parallel Thread to ſeem to cover thoſe two Points already mark'd, and inſtruct an Aſſiſtant to mark out the Projection wanted. Suppoſe, for example, *p h* (*Fig.* 20.) to be that principal Projection. Then to find the Projection (for example *o*) of any other

Point in the Figure to be deſcribed, imagine that Point to be the Vertex of a Triangle, whoſe Baſe is the Original of the Projection already found, and place the Thread (paſſing ſtill through the Point of Sight) in a parallel Situation to the Original of one of the Legs of that Triangle, and caſt its Shadow on the proper Extremity of the Projection given, and mark it as before, and you will have the indefinite Projection of that Leg, (ſuppoſe it to be *h o*.) Do the ſame by the other Leg, and by the Interſection of thoſe Projections (ſuppoſe of *h o* and *p o*) you will have the Projection of the Point ſought. And by this Method may be found the Projections of any Figures whatſoever. I ſhall not enlarge upon this Method, not having had an opportunity of putting it in practice; for which reaſon I only propoſe it as a Hint, which I leave to be further conſider'd of by the Curious.

NUMB. II.

A New THEORY for mixing of COLOURS, *taken from Sir* Iſaac Newton's *Opticks.*

THO' my Deſign was only to treat of *Linear Perſpective*, and not to diſcourſe of all the Parts of *Painting*, yet as this Book will be moſt uſeful to thoſe who practiſe that Art, I thought it would not be improper, nor unentertaining to the Readers, if I took this Occaſion to publiſh ſome Thoughts I have had concerning the Mixture of Colours: which I have fallen into upon conſidering Sir *Iſaac Newton*'s Theory of Light and Colours, in his moſt excellent Treatiſe of *Optics*.

In Colours theſe two things are to be conſider'd, the Hue (which is properly what may be called the Colour,) and the Strength of Light and Shadow. For as different Colours, ſuppoſe Red and Green, may have the ſame Strength of Light; ſo two things, that are one of them much darker than the other, may ſtill have the ſame Hue, as a light Blue and a dark Blue.

With reſpect to the Hue, theſe two things are to be conſider'd, 1*ſt*. The Species of Colour, and 2*dly*. The Perfection and Imperfection of Colour under the ſame Species. Colours differ in Species, as Blue and Red, and Colours of the ſame Species differ in degree of Perfection; as the Red of a Field Poppy is much more perfect than the Red of a Brick. This Quality of Perfection and Imperfection in the Colours, by the Painters is expreſs'd by the Terms Bright, or Clean, or Simple, and Broken; which is taken from their Method of making the imperfect Colours, by the Mixture of other Colours, which is called breaking the Colours. With reſpect to this Quality of Colours, Sir *Iſaac Newton*, in the Book already mention'd, ſhews, that every Ray of Light has its proper Colour, which it always carries with it, and never loſes, in whatever manner it happens to be reflected or refracted. Theſe natural Colours of the Rays are the Bright Simple Colours, and the natural Order of them, as they appear when they are ſeparated by the Refraction of a Priſm, is, Red, Orange, Yellow, Green, Blue, Indico, Violet. All the leſs perfect or broken Colours, are made by the Compoſition and Mixture of theſe ſimple Colours, as Yellow Rays mix'd with Blue Rays, make a Green, but not ſo perfect as the ſimple natural Rays that are Green; and Red and Yellow Rays make an Orange Colour, but not ſo perfect as the natural Orange-colour'd Rays. And by a juſt Proportion of all the Natural Rays together, is produced Whiteneſs, which is indifferent to all the

surely
not!

ſimple Colours, and can't be ſaid to incline more to one Colour than to another. By White I mean any Colour between the lighteſt White and the darkeſt Black; for as we are not now conſidering the Degrees of Light and Shade, all the Colours from Black to White are to be conſider'd as of the ſame Hue.

According to this Obſervation of the Nature of Whiteneſs, it appears that the broken Colours are a Medium between the ſimple Colours and White, and the more broken a Colour is, the nearer it is to White, and the further it is from White, the more ſimple it is.

Having thus explain'd the Nature of the Colours; and the Effect of their Mixture, in order to find exactly what Colour will be produced by the Mixture of any Colours given, Sir *Iſaac* diſpoſes the Colours in the following manner. Let there be a Circle made A D F A, and let the Circumference be divided into ſeven Parts A B, B C, C D, D E, E F, F G, G A, in the ſame proportion to one another as the Fractions $\frac{1}{9}, \frac{1}{16}, \frac{1}{10}, \frac{1}{9}, \frac{1}{16}, \frac{1}{16}, \frac{1}{9}$; which are the Proportions of the Muſical Notes *Sol, la, fa, ſol, la, mi, fa, ſol.* Between A and B place all the Kinds of Red, from B to C place all the Kinds of Orange, from C to D place all the Kinds of Yellow, from D to E place all the Kinds of Green, from E to F place all the Kinds of Blue, from F to G place all the Kinds of Indico, and from G to A place all the Kinds of Violet. Having thus diſpoſed the ſimple Colours, the Center of the Circle O will be the Place of White. And between the Center and the Circumference are

Fig 25

are the Places of all the broken compounded Colours, thoſe neareſt the Center being the moſt compounded, and thoſe fartheſt from it being the leaſt compounded. As in the Line O 1, all the Colours at 1, 2, 3, 4, are of the ſame Species, that is, Green inclining towards Blue, but the Colour at 1 is the ſimple natural Colour; that at 2 is ſomething compounded, or broken; that at 3 is more broken; and that at 4 is ſtill more broken.

The Colours being thus diſpoſed, to know what Colour reſults from the Mixture of any Colours given, find the Center of Gravity of the Places of the Colours given, and that will ſhew the Character of the Compound. For example, ſuppoſe I would know what Colour would reſult from the Mixture of two Parts of the ſimple Yellow at P, with three Parts of the ſimple Blue at Q: I find the Center of Gravity 3 of the Points P and Q; that is, I draw P Q, and having divided it into five Parts (which is the Sum of three and two) I take the Point 3 three Parts from P (becauſe there are three Parts of Blue) and two Parts from Q, (becauſe there are two Parts of the Colour at P.) Then drawing O 3 cutting the Circumference in 1, by the Place of the Point 1, (which is between D and E, but nearer to E) I find the Mixture is a Green inclining towards Blue; but becauſe 3 is near the Middle between the Center and the Circumference, the Colour is pretty much broken. To make the ſame thing more clear by another Example; ſuppoſe I would know what would reſult from a Mixture of two

Parts Yellow at P, three Parts Blue at Q, and five Parts Red at R. Firſt I find the Place 3 of the Mixture of the Yellow and the Blue, as before. Then drawing the Line 3 R (becauſe there are five Parts of the Colour at 3, and five Parts of the Colour at R) I divide it into ten Parts, and take the Point *r* five Parts diſtant from R. By this means *r* is the Center of Gravity of the three Colours at P, Q, and R, and is conſequently the Place of the Mixture; which by drawing O *r* cutting the Circumference in *s*, I find to be an Orange a little inclining towards Red, and becauſe *r* is much nearer the Center than the Circumference, the Colour is very much broken. And thus one may proceed in other Caſes.

Again, having given the Place of any compound Colour, one may find what Colours may be mix'd to compound it. Thus having given the Colour at 3, drawing any Line P 3 Q thro' 3, the Colour propoſed may be made by a Mixture of the Colours in P and Q, taking ſuch a Proportion of them as is expreſſed by the Lines 3 P and 3 Q, that is, taking of the Colour P as much as in proportion to 3 Q, and as much of the Colour Q as is in proportion to 3 P. Or having drawn O 3 paſſing thro' the Points 1, 2, 4, the ſame Colour may be produced by mixing the Colours in 2 and 4 in proportion to the Lines 4.3 and 2.3; or it may be produced by breaking the ſimple Colour at 1 with White (which is at O) in the proportion of the Lines 3.1 and 3 O. And thus in other Caſes.

The Proportions hitherto mention'd of the Colours to be uſed in the Mixtures, relate to the Quantity of the Rays of Light, and not to the Materials which artificial Colours are made of. Wherefore if ſeveral artificial Colours were to be mix'd according to theſe Rules, and ſome of them are darker than others, there muſt be a greater Proportion uſed of the darker Materials, to produce the Hue propoſed, becauſe they reflect fewer Rays of Light in proportion to their Quantities; and a leſſer Proportion muſt be uſed of the lighter Materials, becauſe they reflect a greater Quantity of Light.

If the Nature of the material Colours, which are uſed in Painting, was ſo perfectly known, as that one could tell exactly what Species of Colour, how perfect, and what degree of Light and Shade each Material has with reſpect to its Quantity, by theſe Rules one might exactly produce any Colour propoſed, by mixing the ſeveral Materials in their juſt Proportions. But tho' theſe Particulars cannot be known to ſufficient Exactneſs for this purpoſe, beſides the Tediouſneſs that would be in Practice, to meaſure the Colours according to their exact Proportions; yet the Knowledge of this Theory may be of great uſe in Painting. Suppoſe, for example, I had a Palate provided with the ſeveral Colours at *a*, *b*, *c*, *d*, *e*: Suppoſe for inſtance at *a* Carmine; at *b* Orpiment; at *c* Pink; at *d* Ultramarine at *e* Smalts; and I had occaſion to make a broken Green, ſuch as I judge ſhould be placed at *x*. Looking round the Point *x*, I ſee that it does not lie a great deal out of a Line drawn thro' *c* and *d*; therefore I conclude, that mixing the Colours *c* and *d* will come very near what I want. But becauſe *x* is nearer to the Center O than the Line *c d*, having brought my Tint as near as I can to what I want, ſuppoſe to *z*, I look from *z* croſs *x* for ſome Colour oppoſite to *z*, to break the Tint with, and I find the neareſt to be *a*, therefore by mixing of the Colour *a* I bring the Compoſition to the Tint I have occaſion for. If the Colour *a* carries the Tint too much towards the Line O D, I put a little more of the Colour *d*, which brings it into the right place. Or having got the Tint *z*, I might have broken it with White, whoſe place is at the Center O. Or putting a greater proportion of the Colour *d* inſtead of *a*, I may afterwards break the Tint by means of the Colour *b*. And in the ſame manner by only inſpecting this Scheme, one may ſee in what manner to make any Tints whatſoever, that can be produced by the Colours that one uſes. Thus one ſees that Red and Yellow makes a broken Orange Colour, which may ſtill be more broken by adding Blue, or Indico, or Violet, which are to be taken one or other, as one would have the Tint inclined more to the Yellow or to the Red; Blue bringing it toward the Yellow, and breaking it much; and Violet carrying it towards the Red, and not breaking it ſo much.

From theſe Principles one may ſee the Reaſon why the Materials of the brighteſt and ſimpleſt Colours are the moſt valuable, and of them why the lighteſt are moſt to be eſteem'd. The ſimpleſt

Colours are the moſt valuable, becauſe they cannot be produced by Mixture; for Mixture always breaks the Colours. Suppoſe *a*, *b*, *c*, *d*, *e*, to be all the Colours you have, then drawing Lines to join the Points, *a*, *b*, *c*, *d*, *e*, all the Tints that can be produced by thoſe Colours will have their Places within the Area of the Polygone *a b c d e*. That the lighter Colours are more valuable than the dark ones, is becauſe Black does not break the Colours ſo much as White; ſo that it is eaſier to make the clean dark Tints with light Colours and Black, than to make the bright light ones, with dark Colours and White. For by what has been ſhew'd, White breaks the Colours very much, but Black being nothing but the abſence of Light, only darkens the Colours. Tho' upon account of the Imperfection of the Materials that are in uſe, Black does alſo break the Colours ſomething, becauſe there is no Material ſo perfectly black as to have no Colour at all, as one may ſee by the beſt Blacks having Lights and Shades. There will be other Exceptions alſo to be made in the application of theſe Obſervations to Practice, upon account of the particular Qualities of the Materials ſome Colours are made of. If all the Colours were as dry Powders, which have no effect upon one another, when mix'd, theſe Obſervations would exactly take place in the mixing of them. But ſome Colours are of ſuch a Nature, that they produce a very different effect upon their Mixture, to what one would expect from theſe Principles. So that it is poſſible there may be ſome dark Materials, which when diluted with White, may produce cleaner and leſs compounded Colours than they gave when ſingle; as ſome Colours do very well to glaze with, which don't look well laid on in a Body. But theſe Properties of particular Materials I leave to be conſider'd by the Practitioners in this Art.

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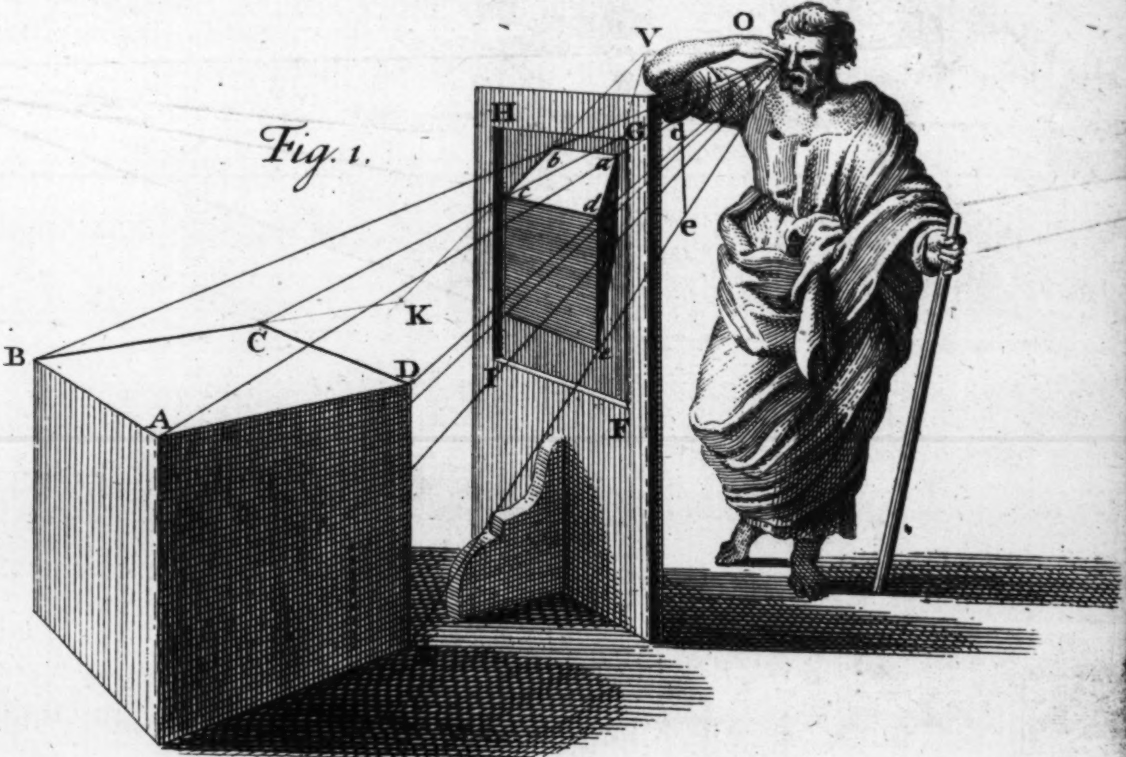

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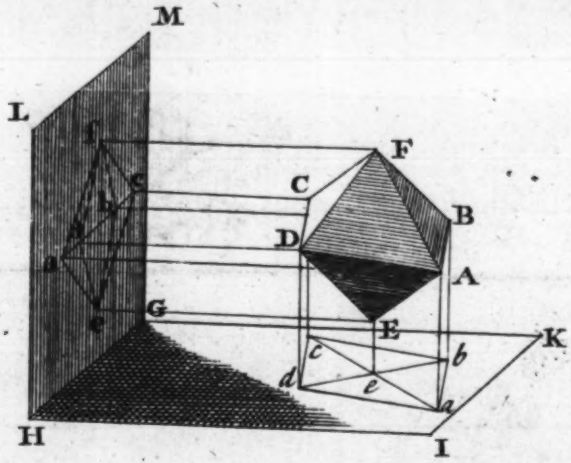

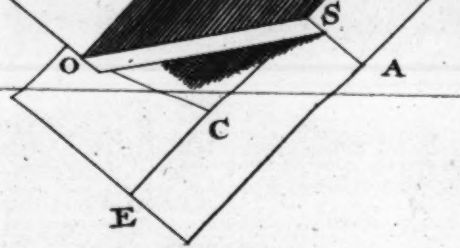

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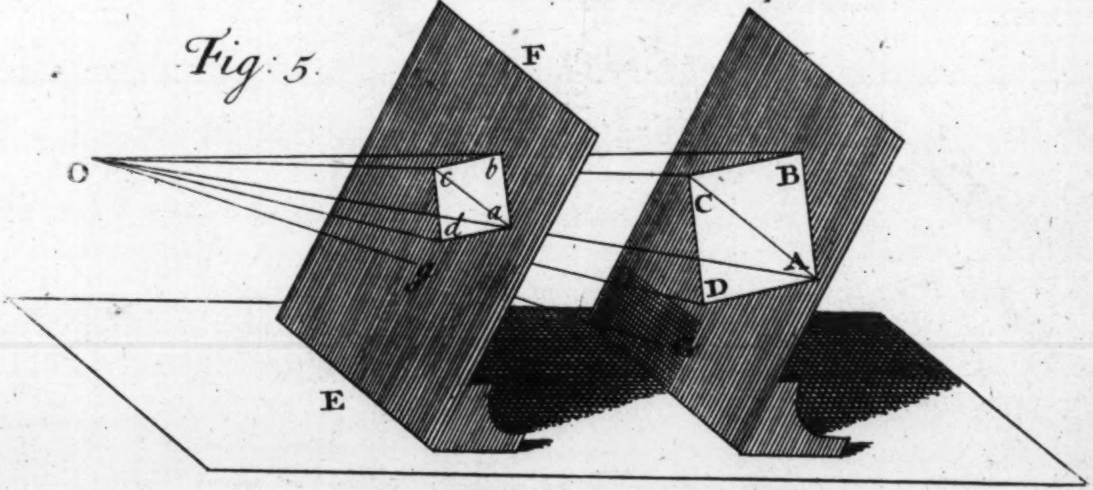

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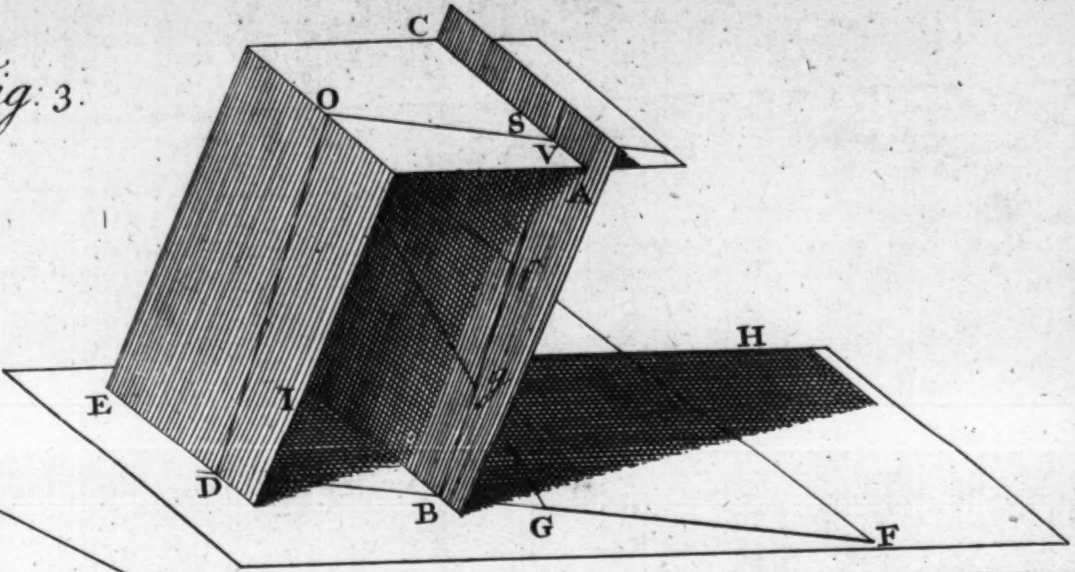

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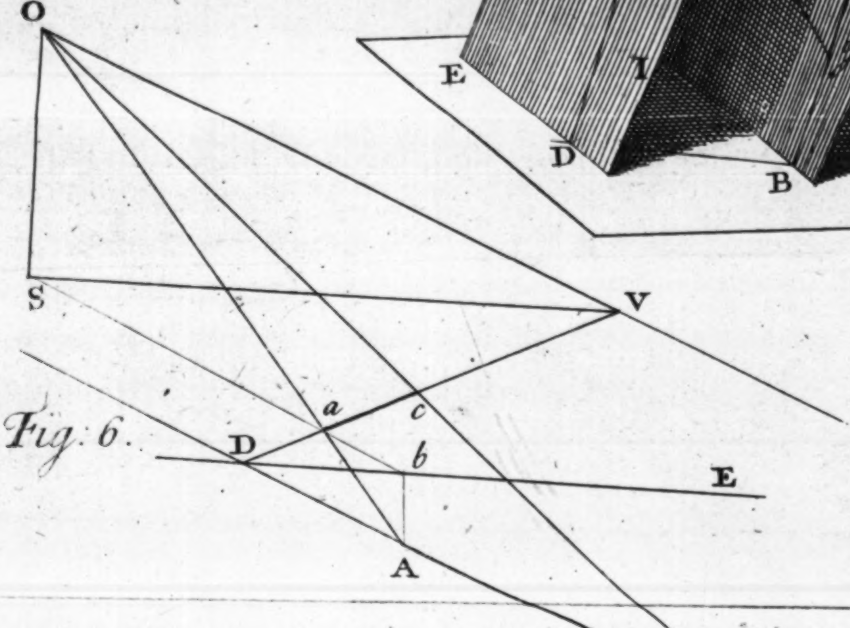

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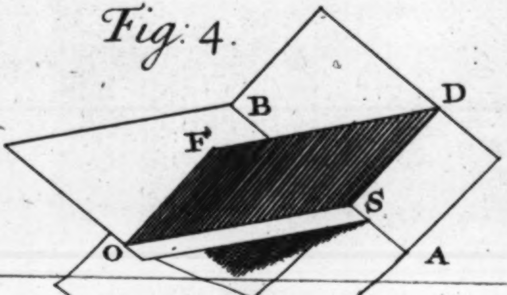

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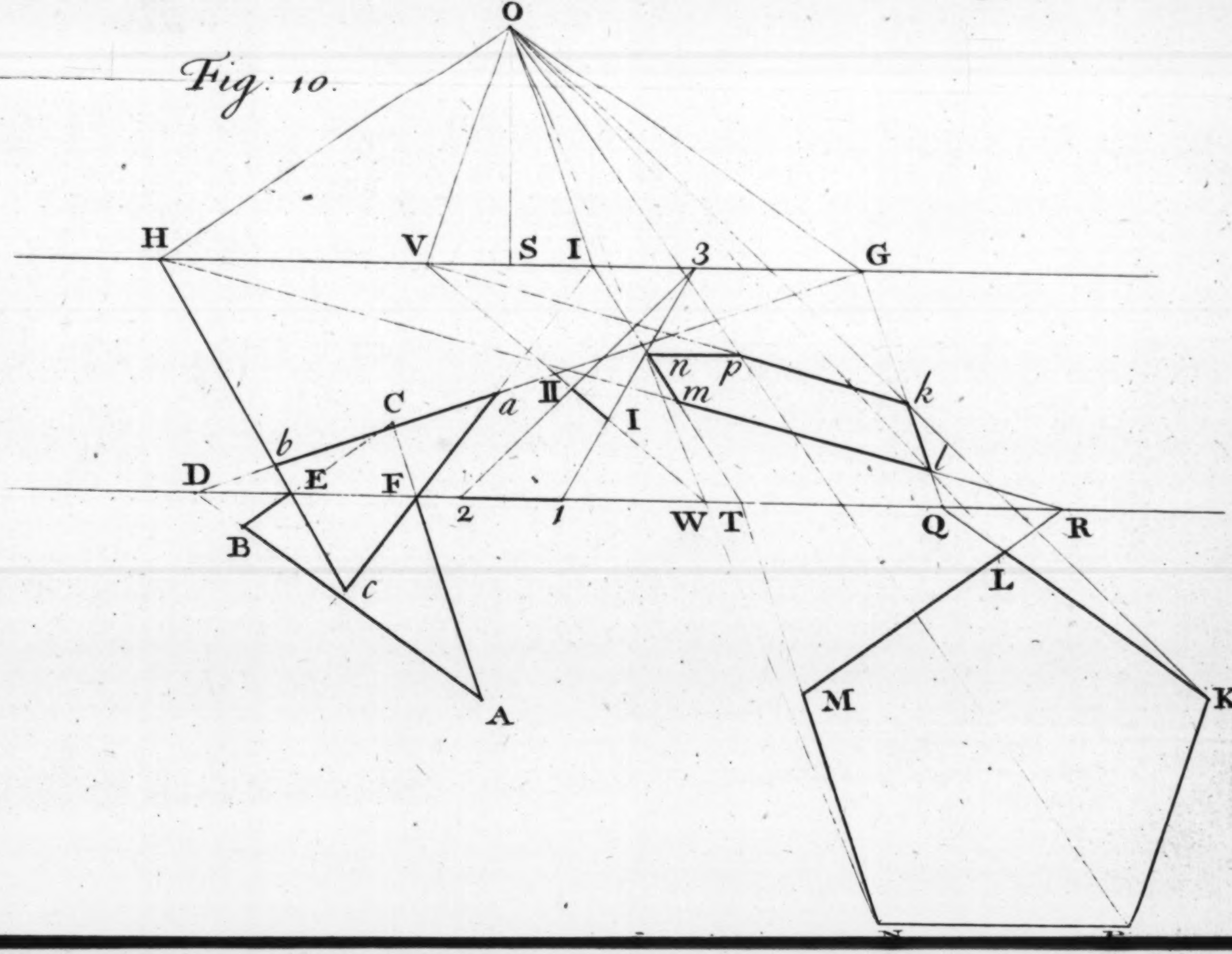

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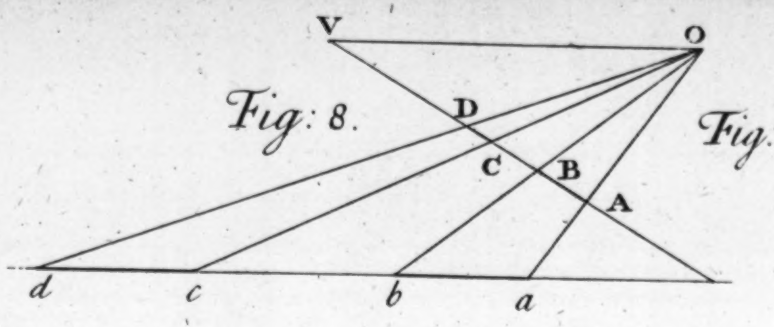

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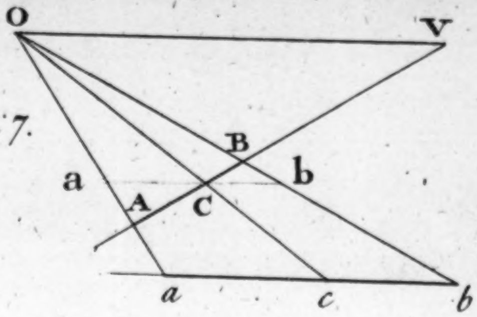

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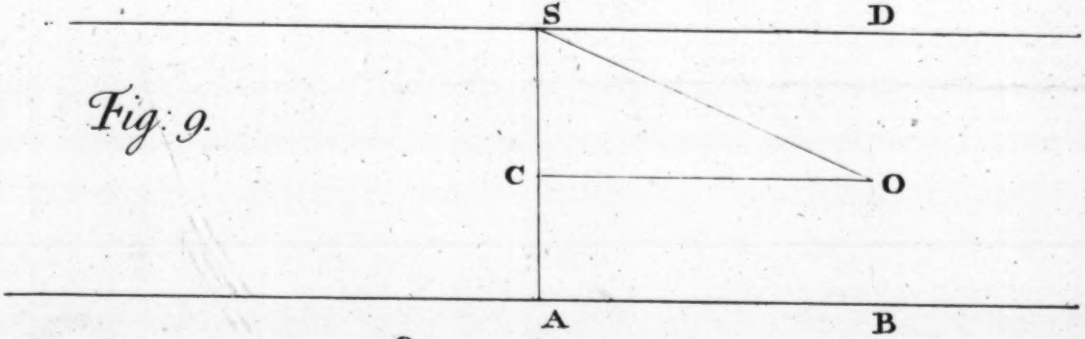

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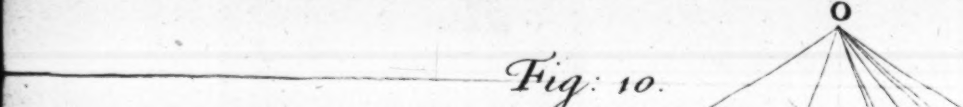

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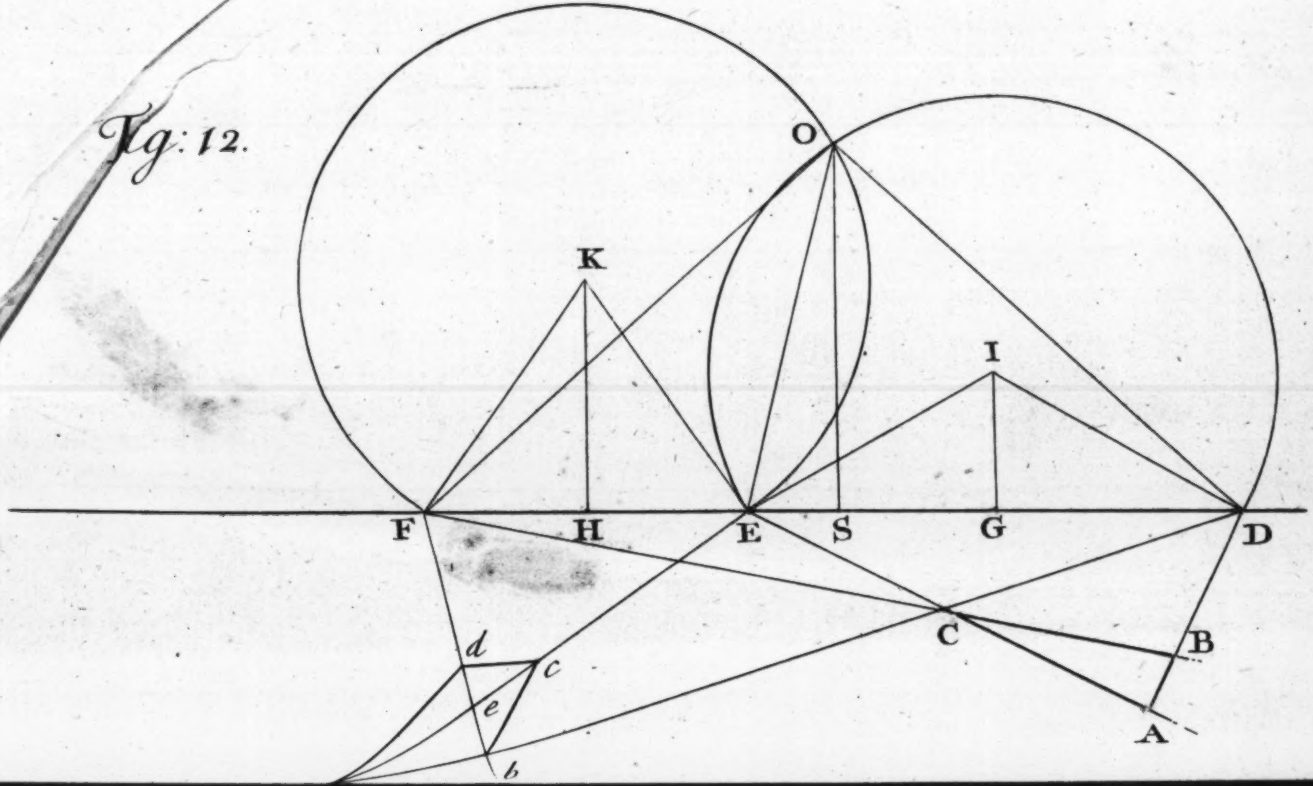

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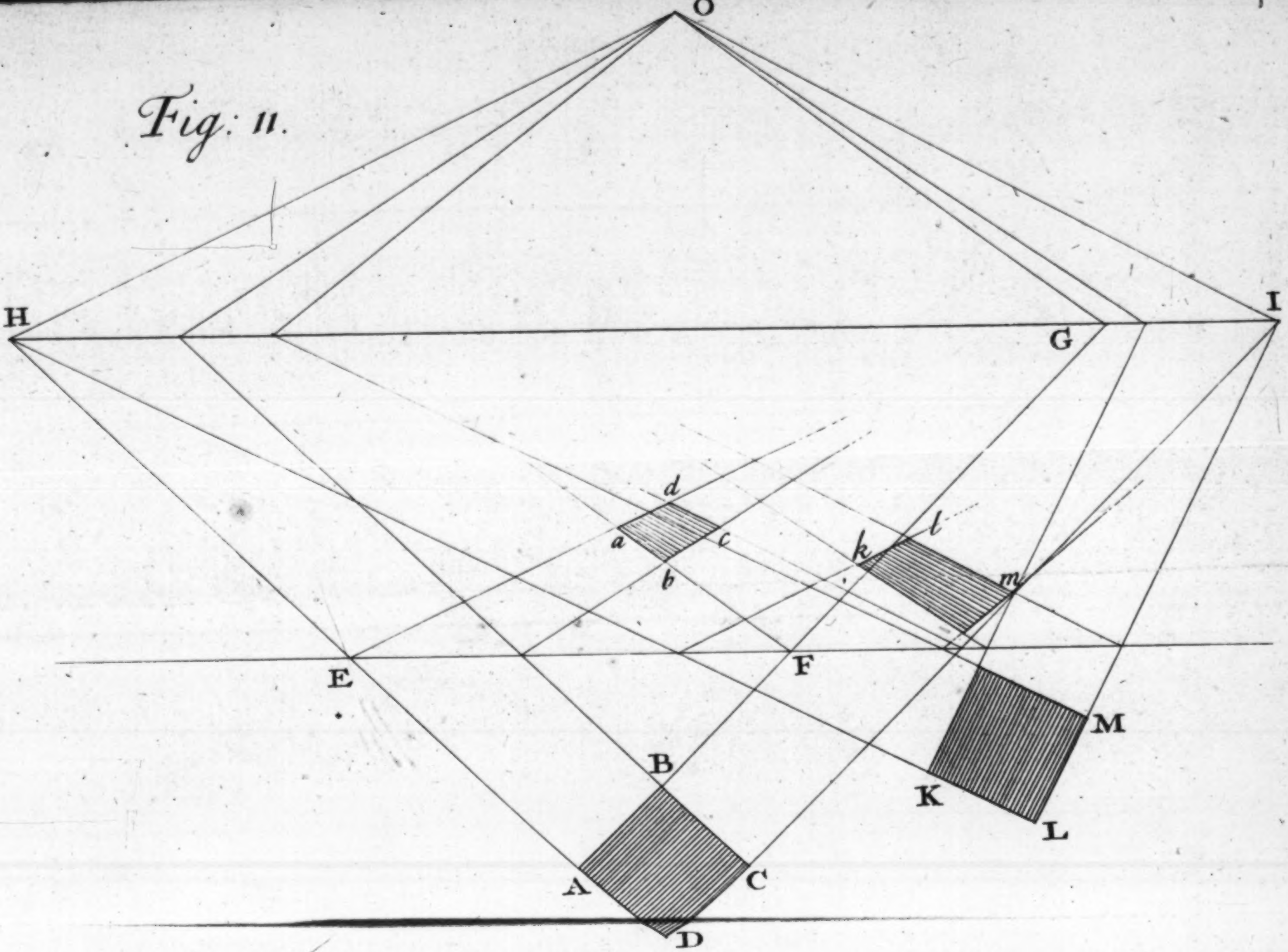

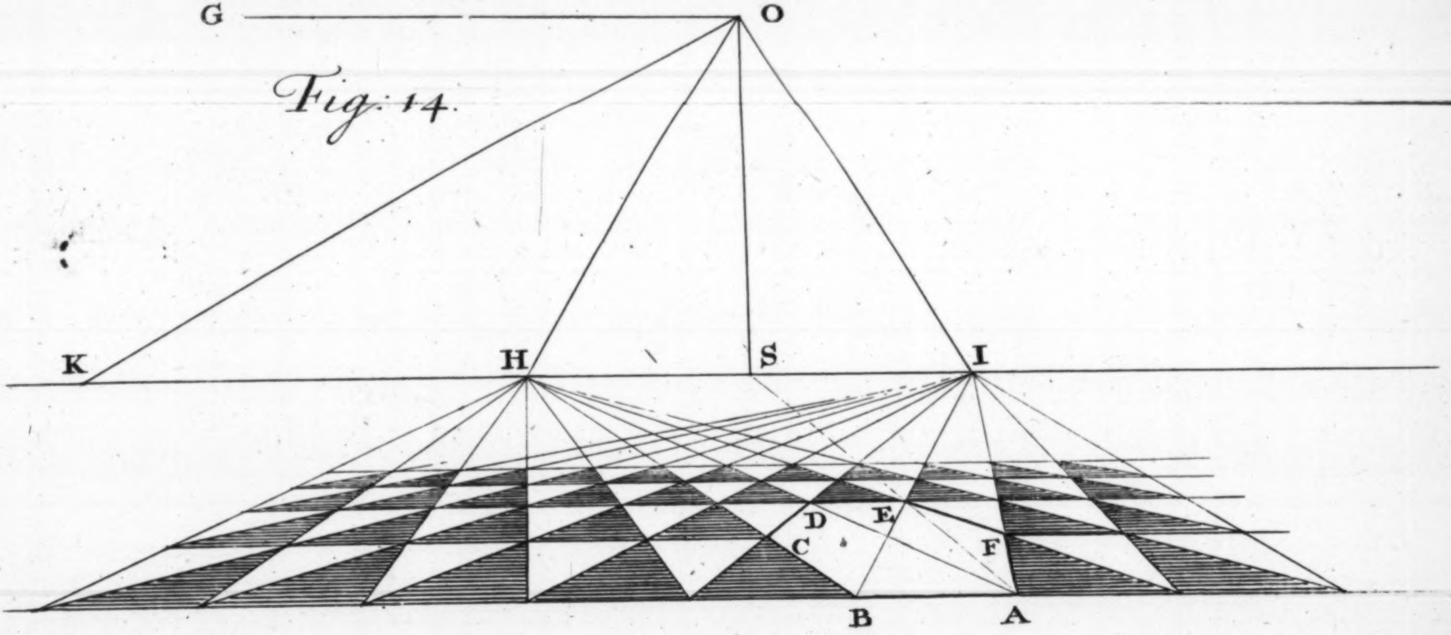

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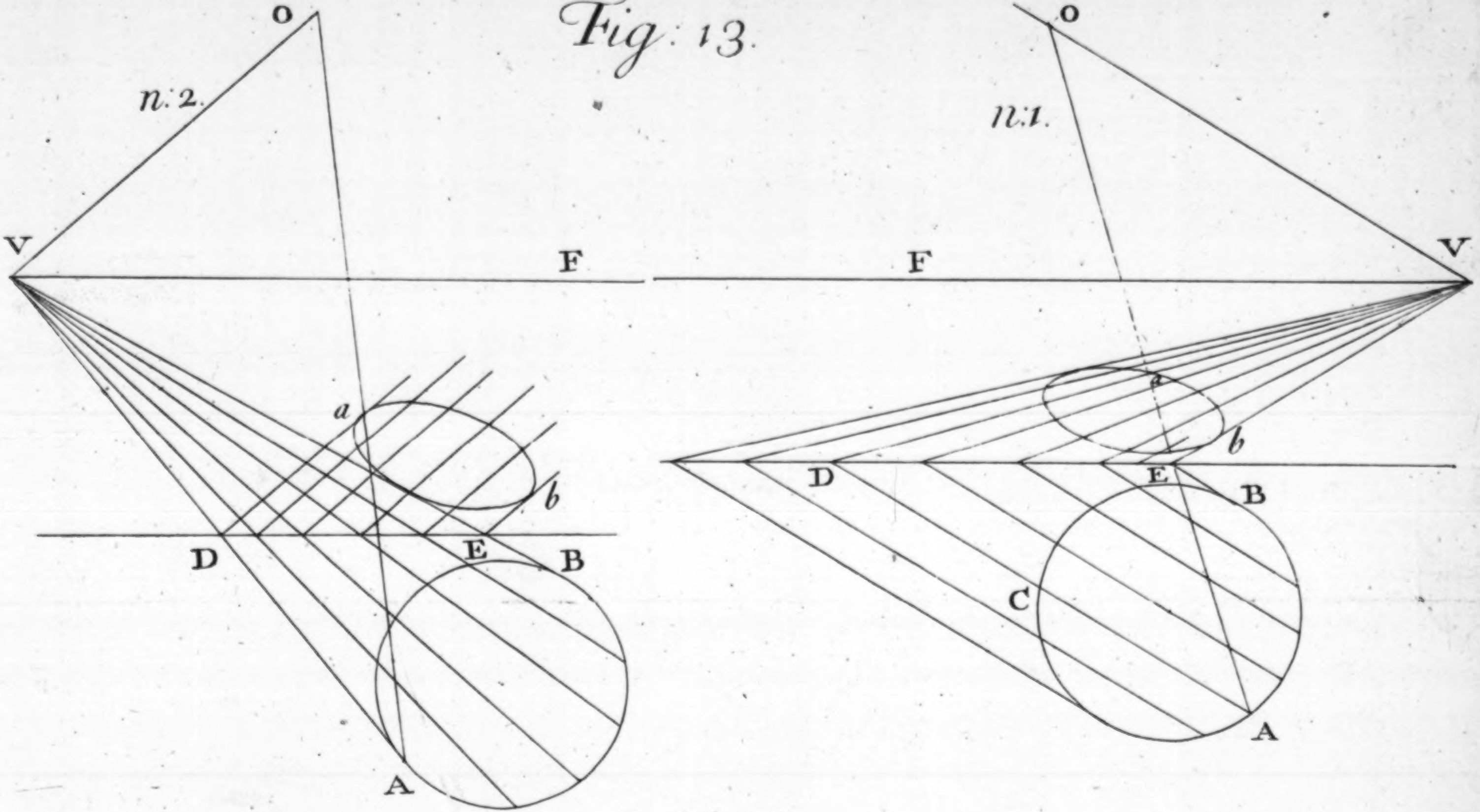

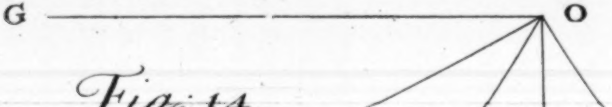

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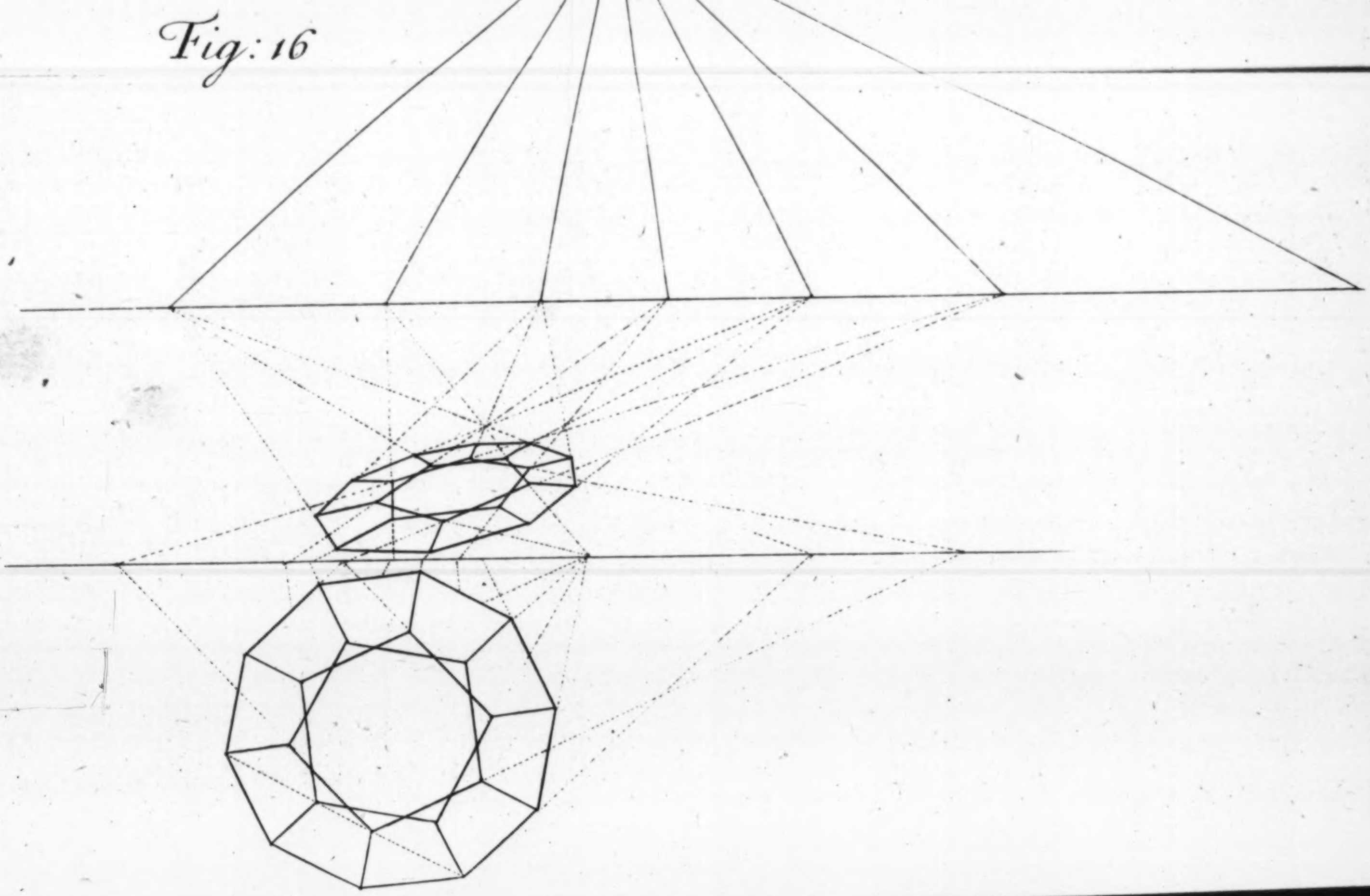

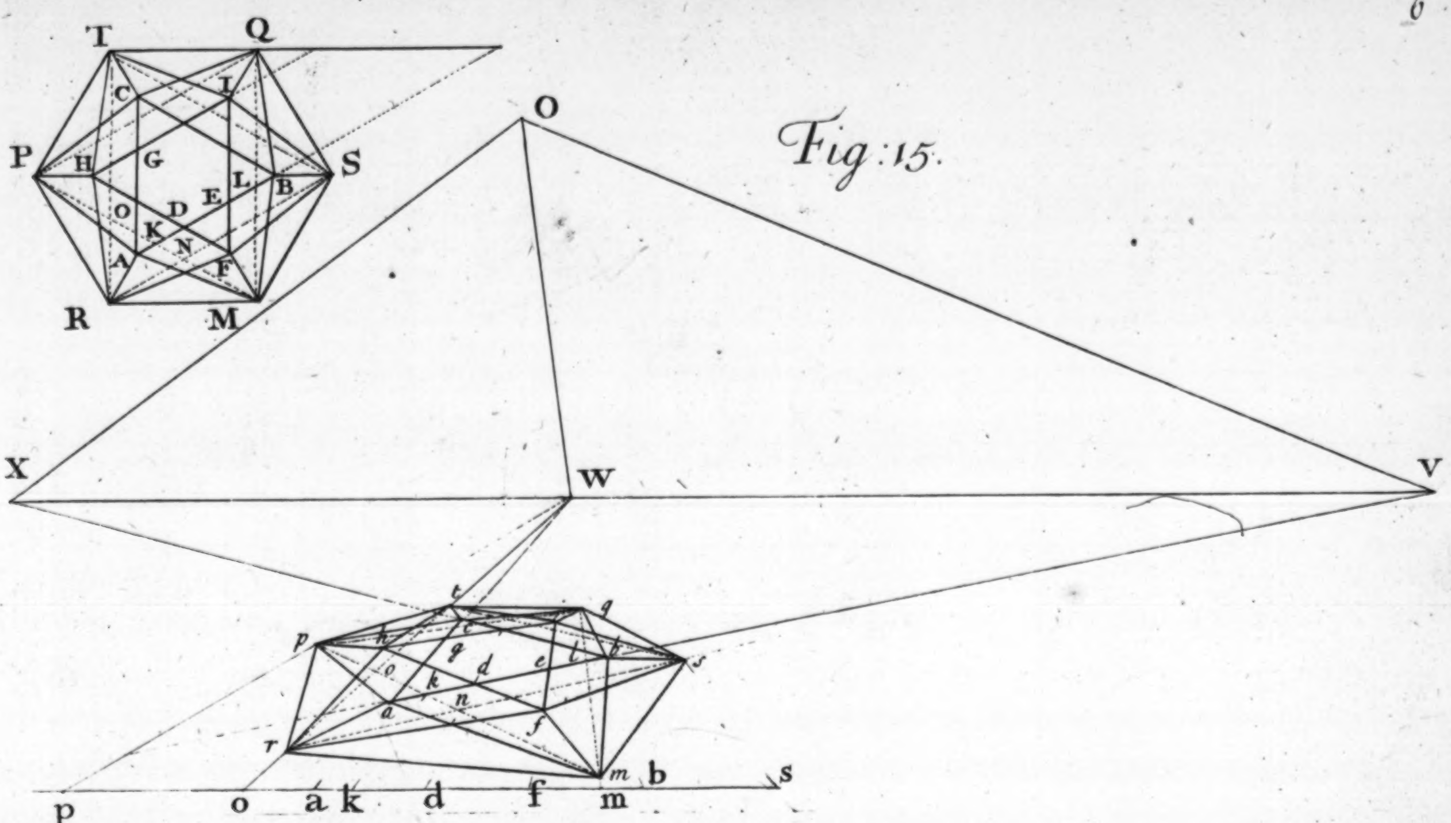
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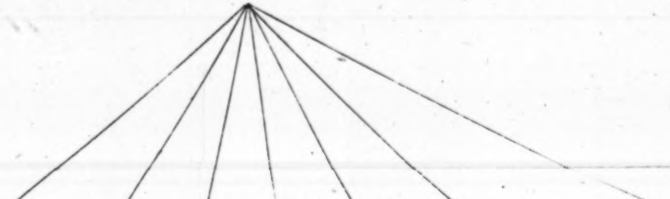

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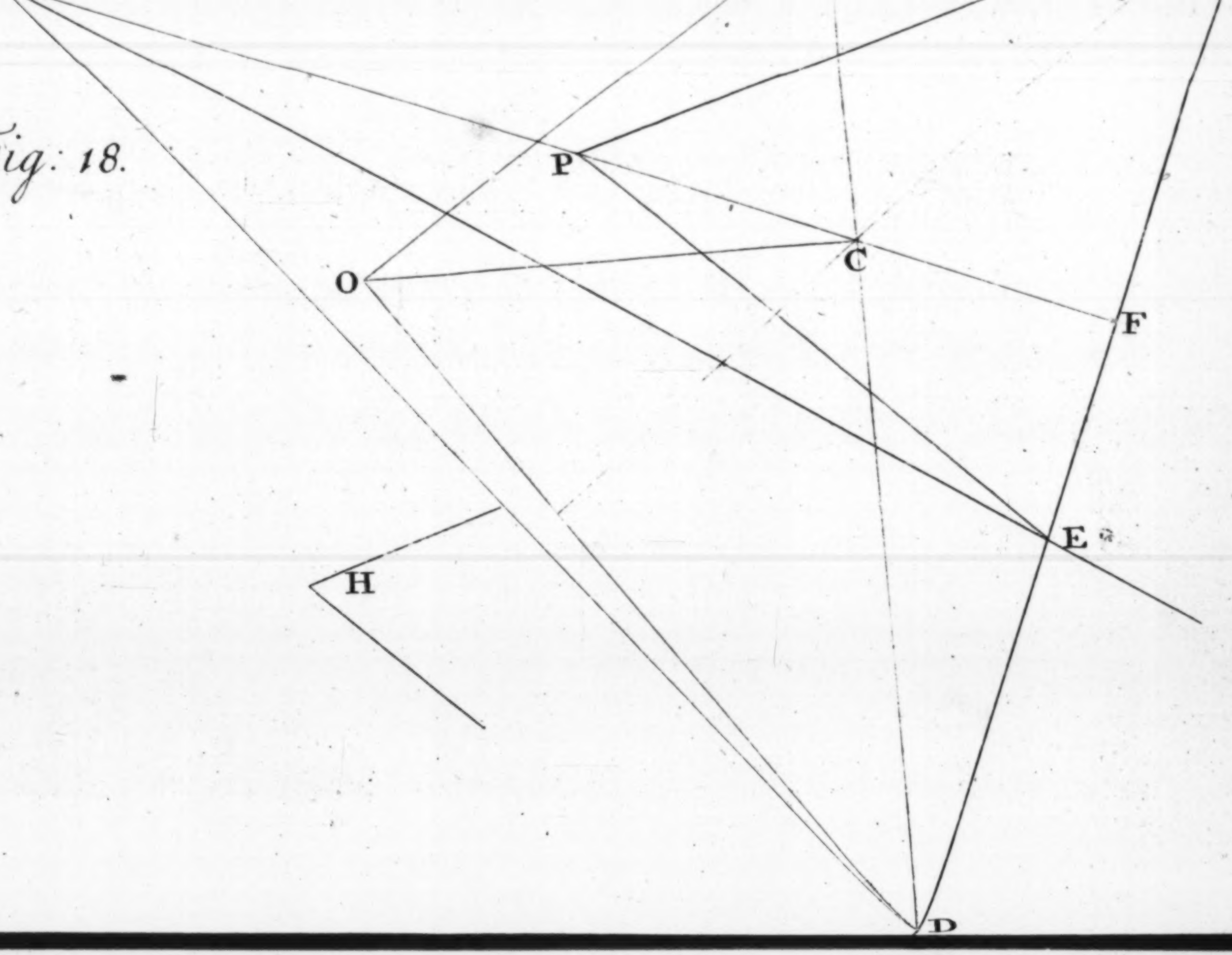

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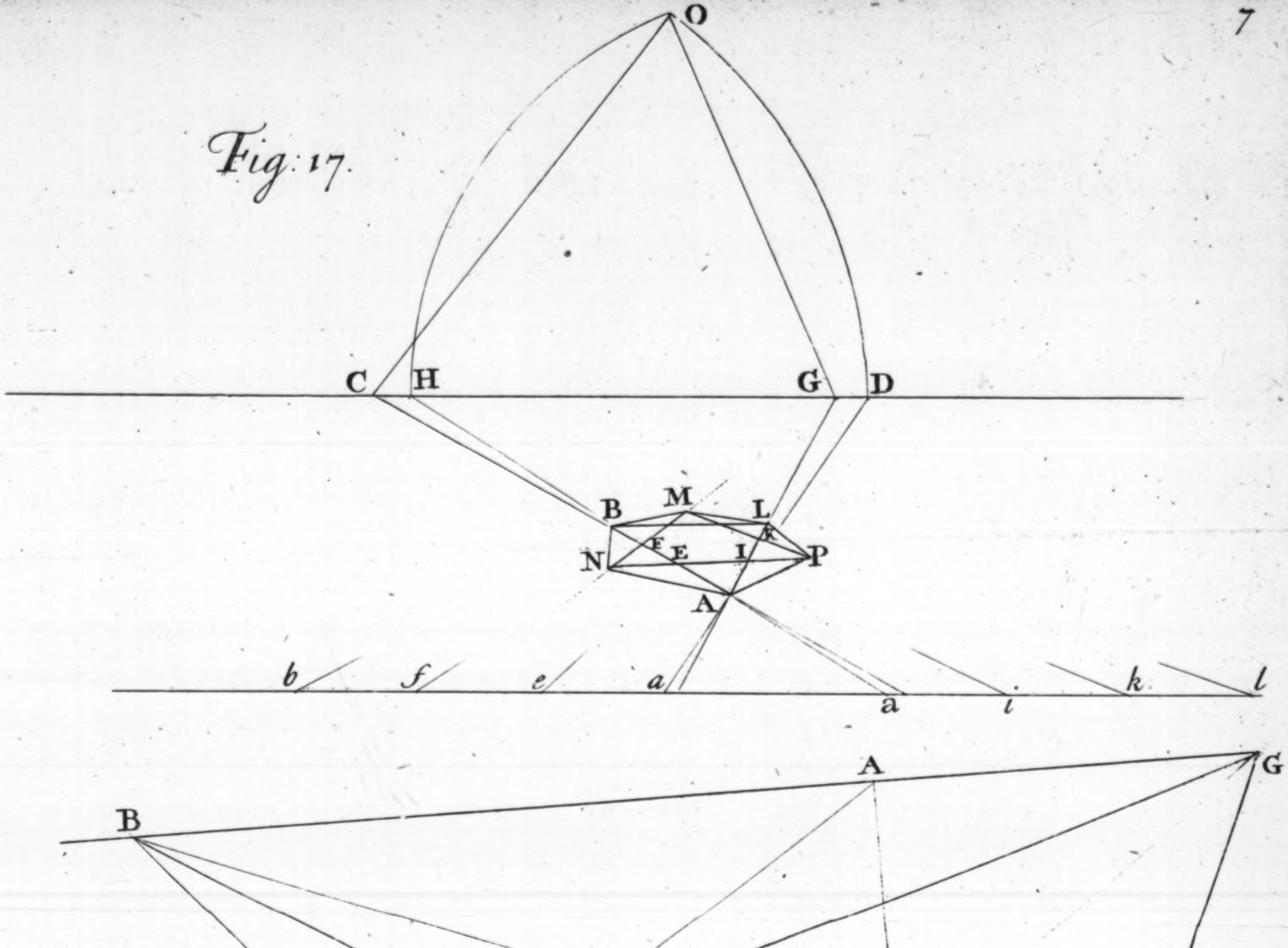

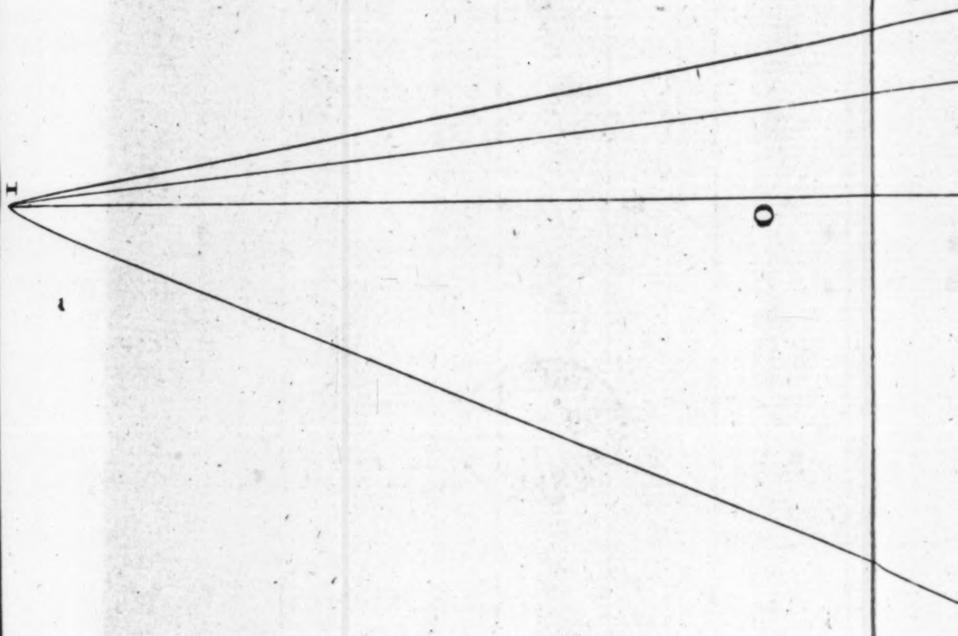
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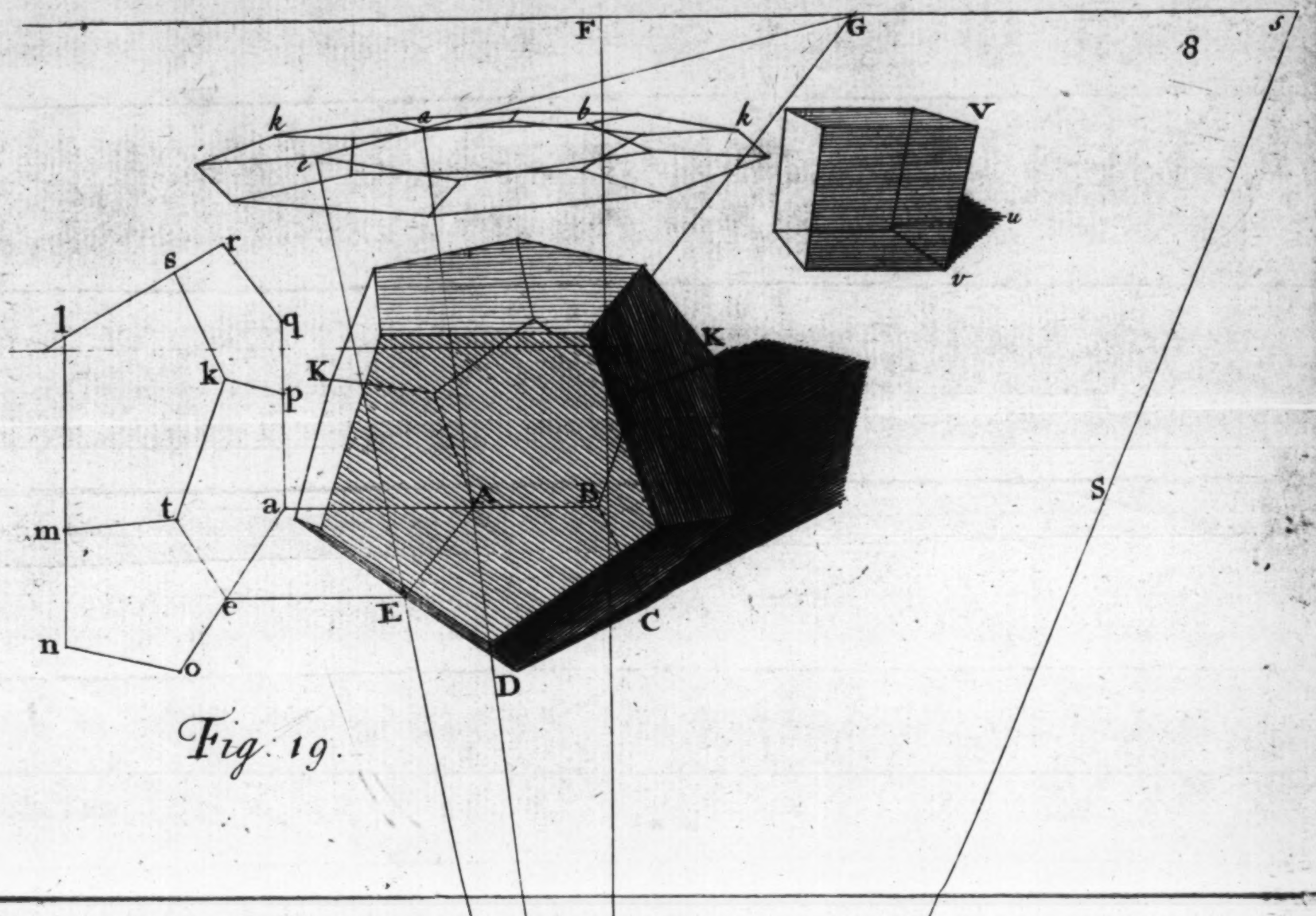

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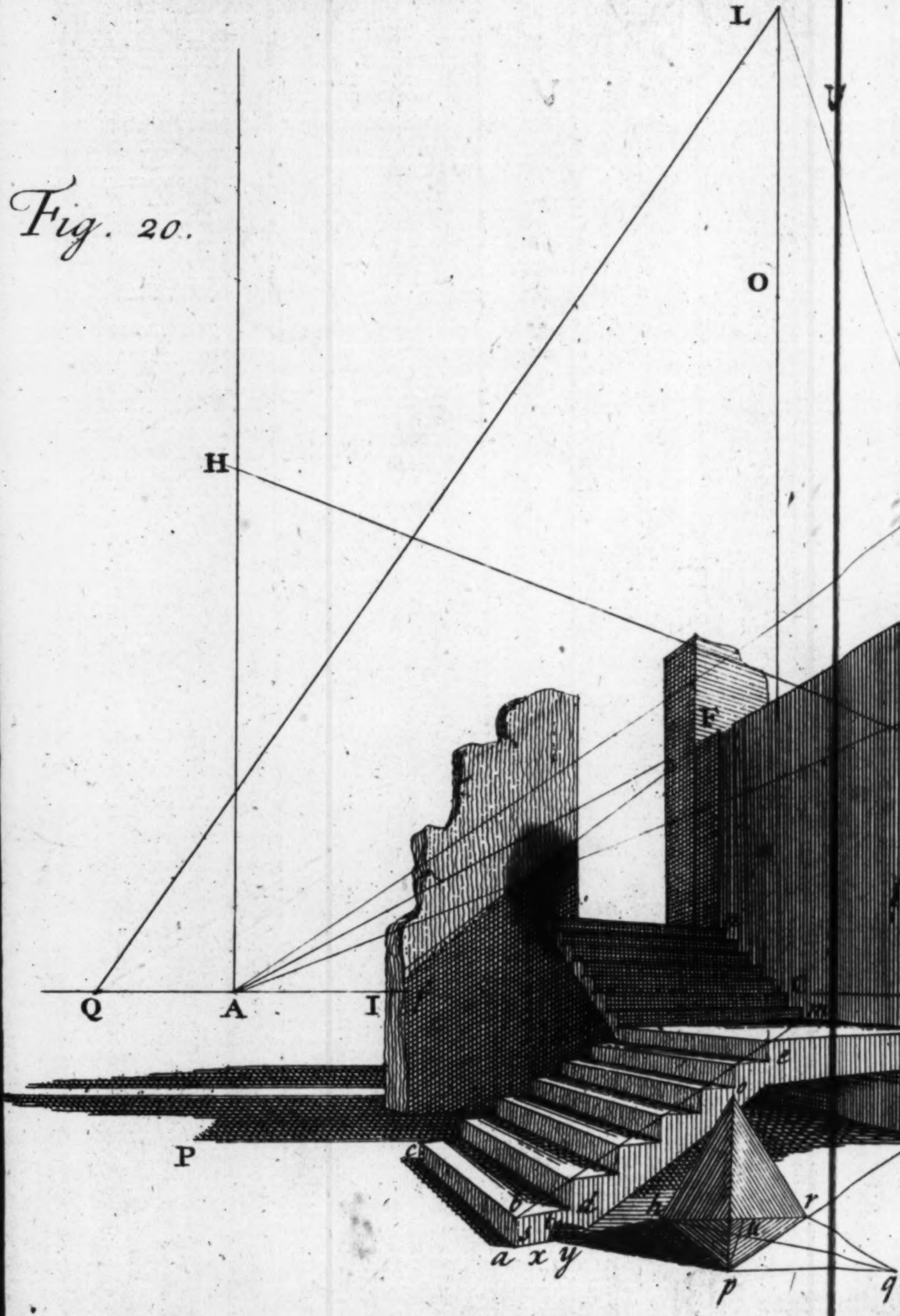

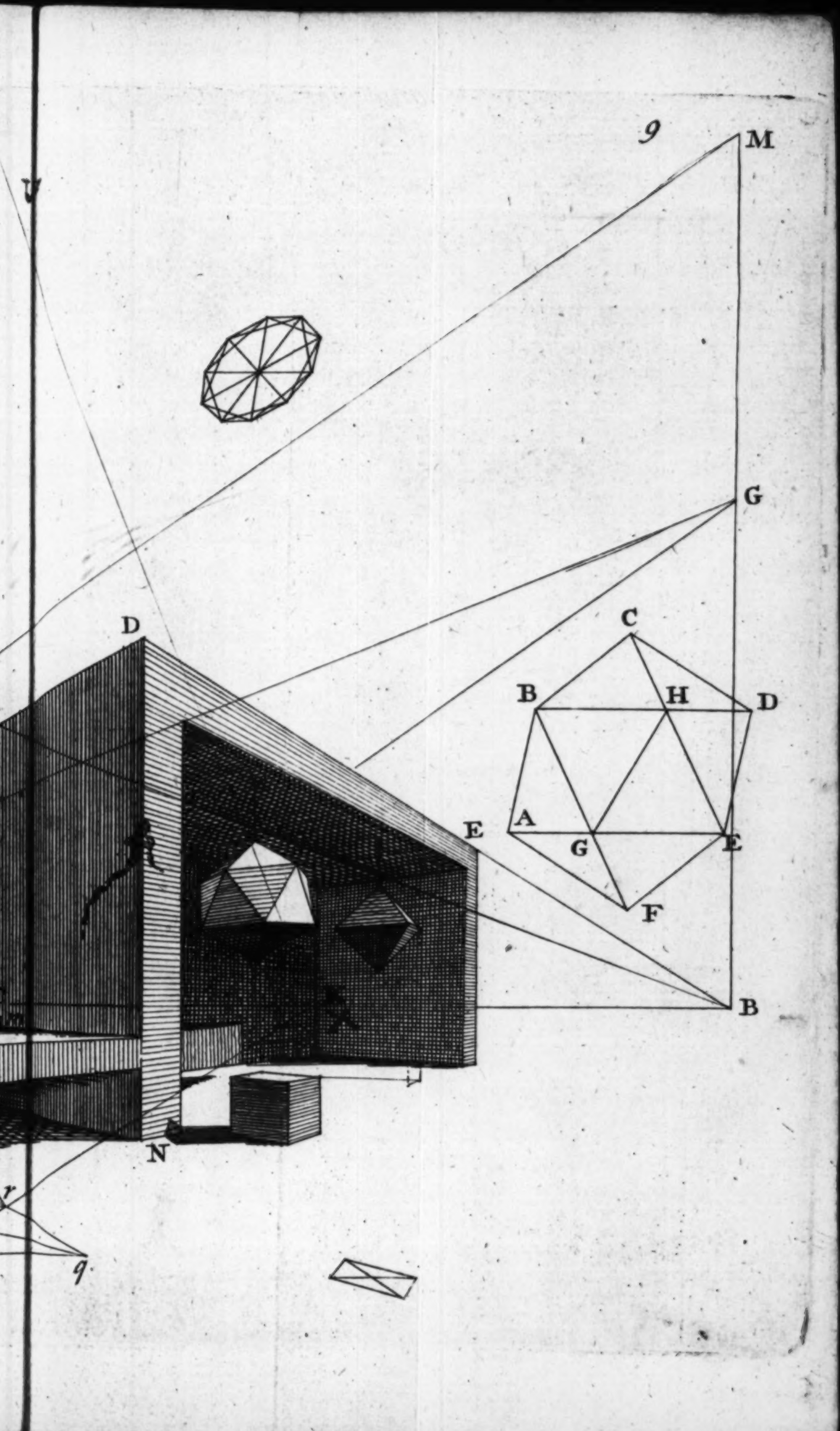

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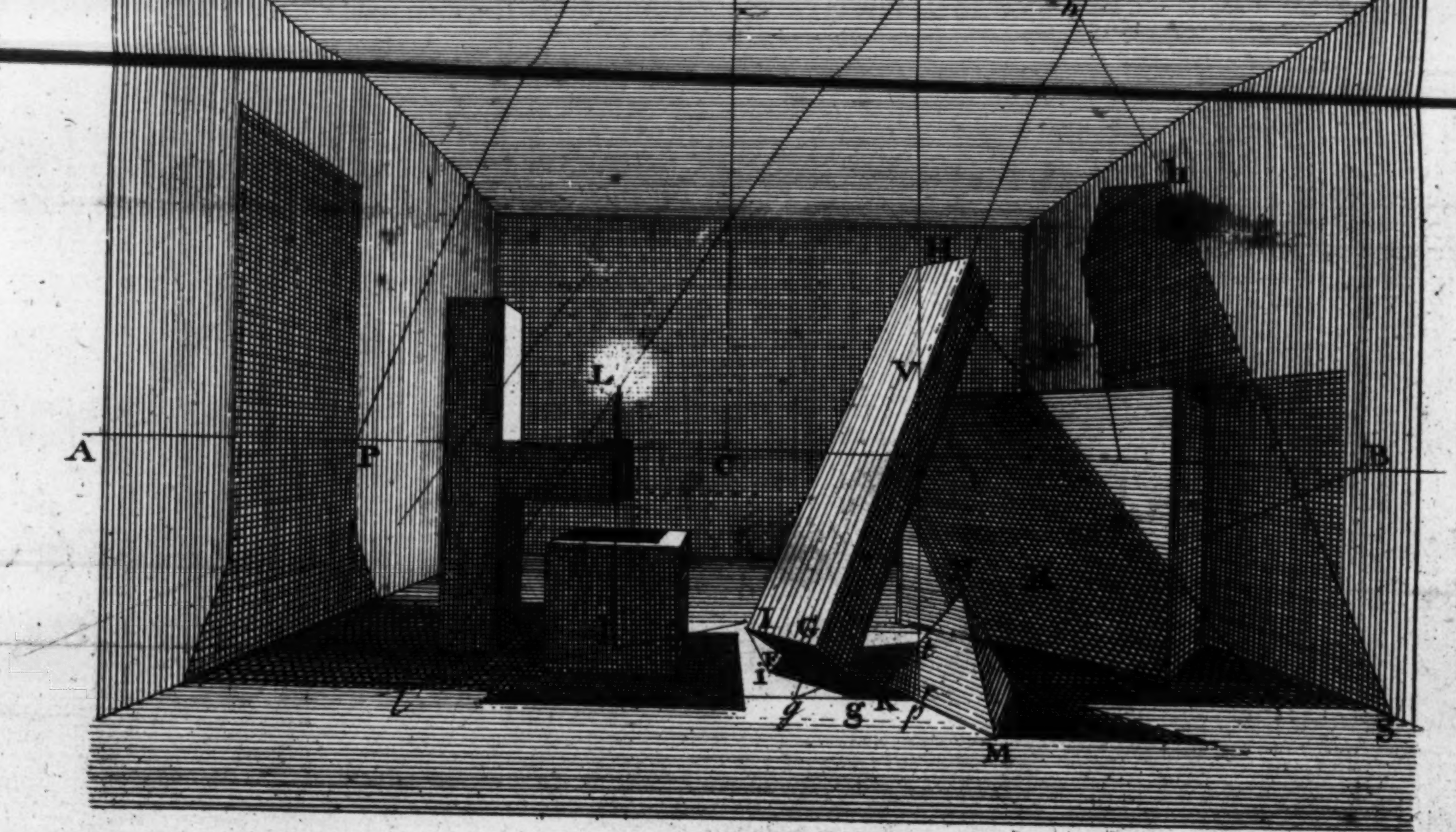
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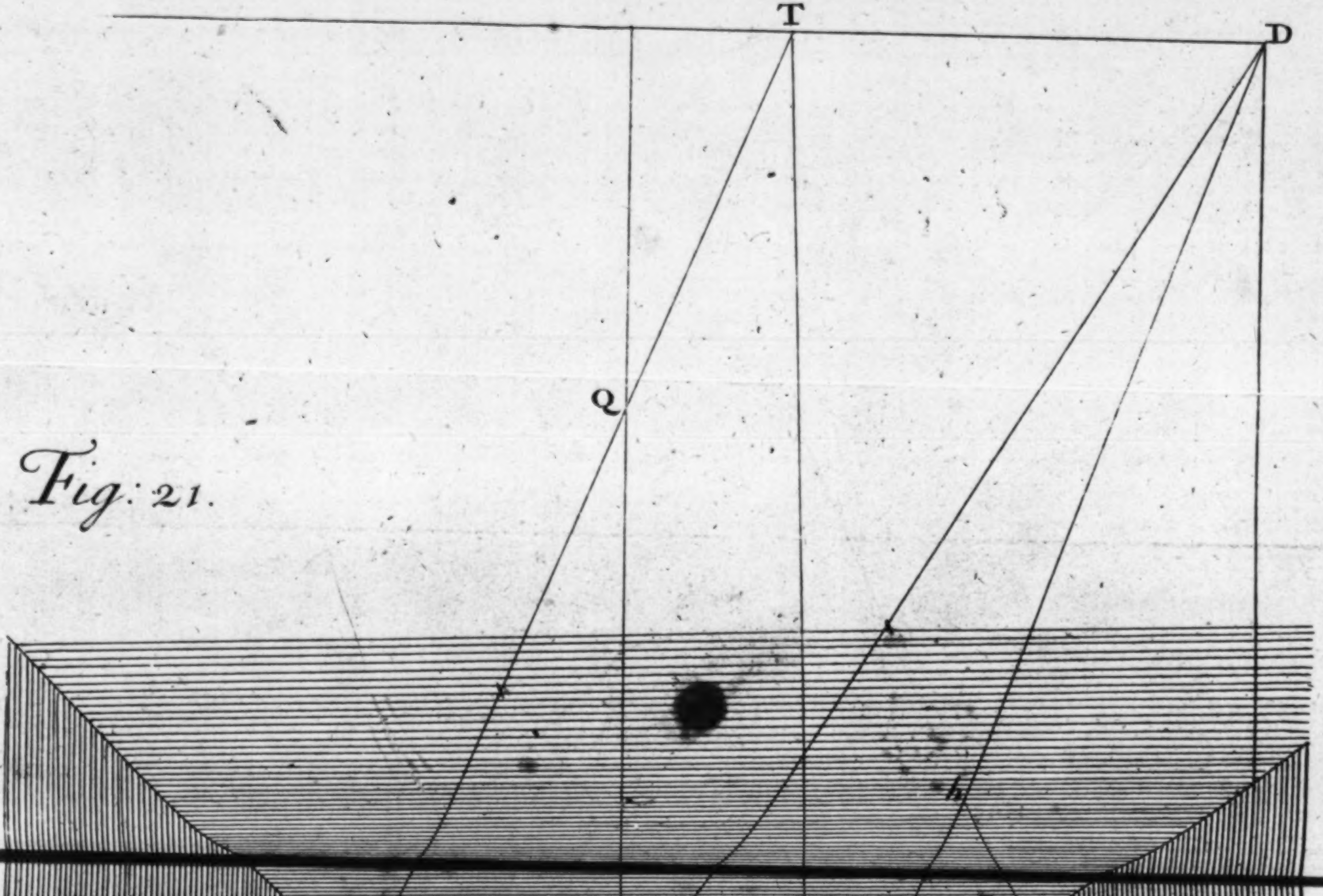

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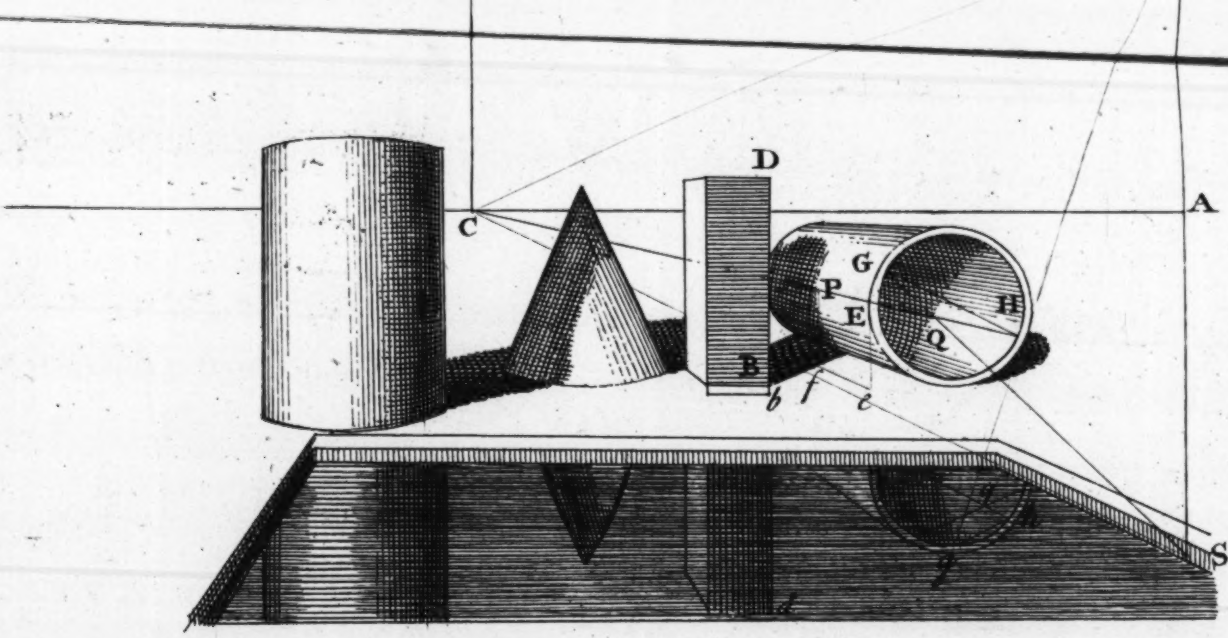
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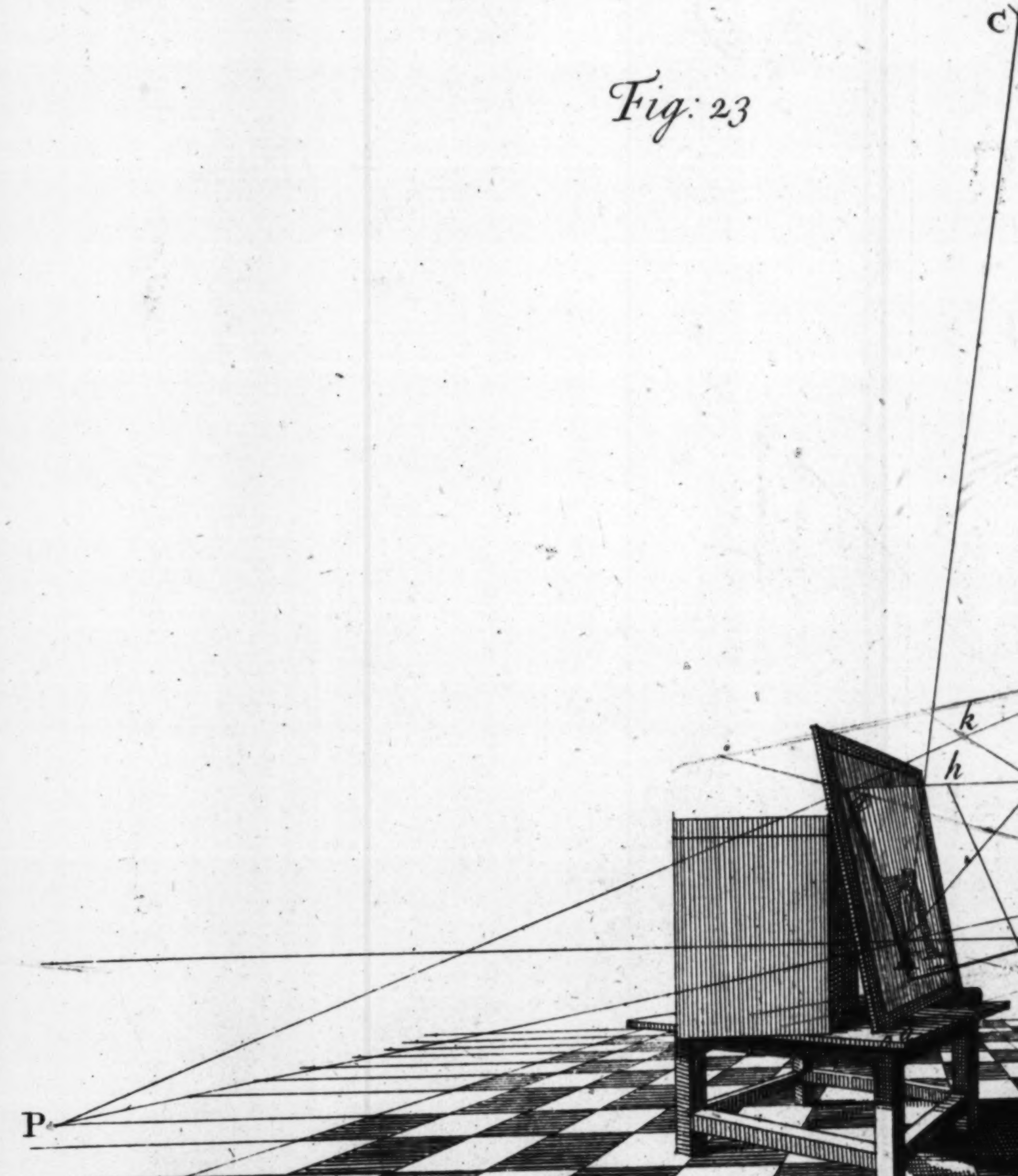

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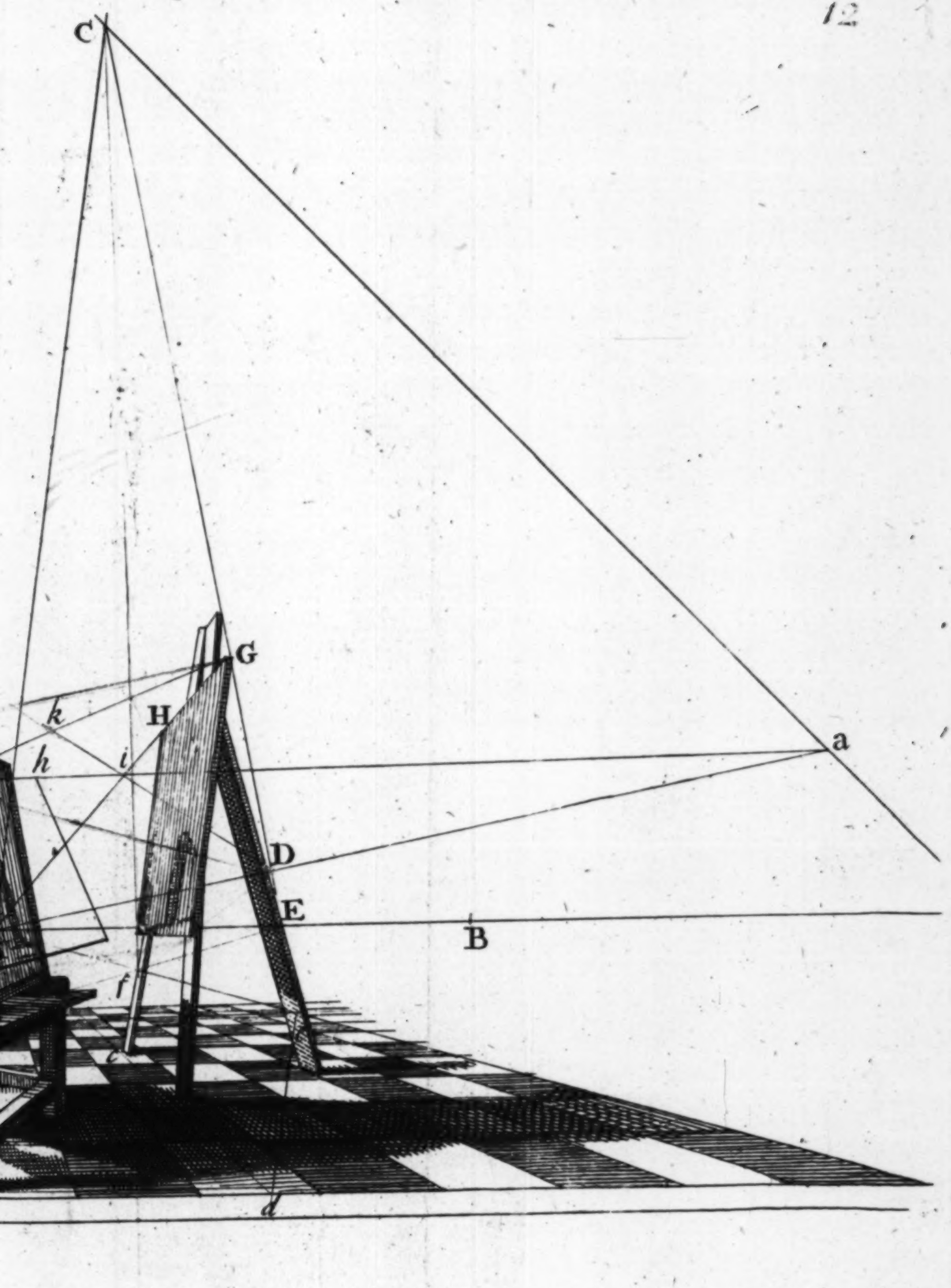

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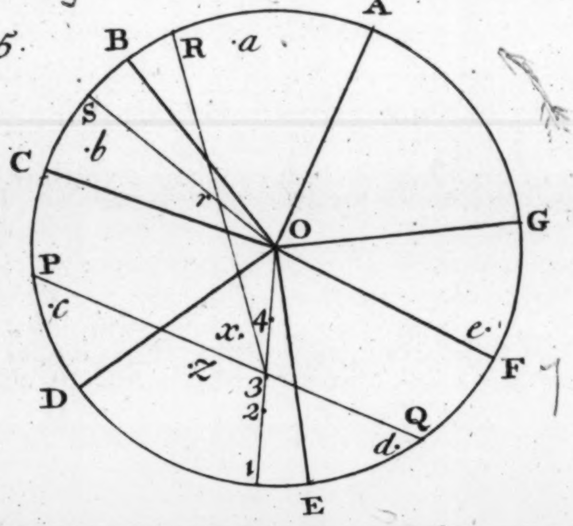

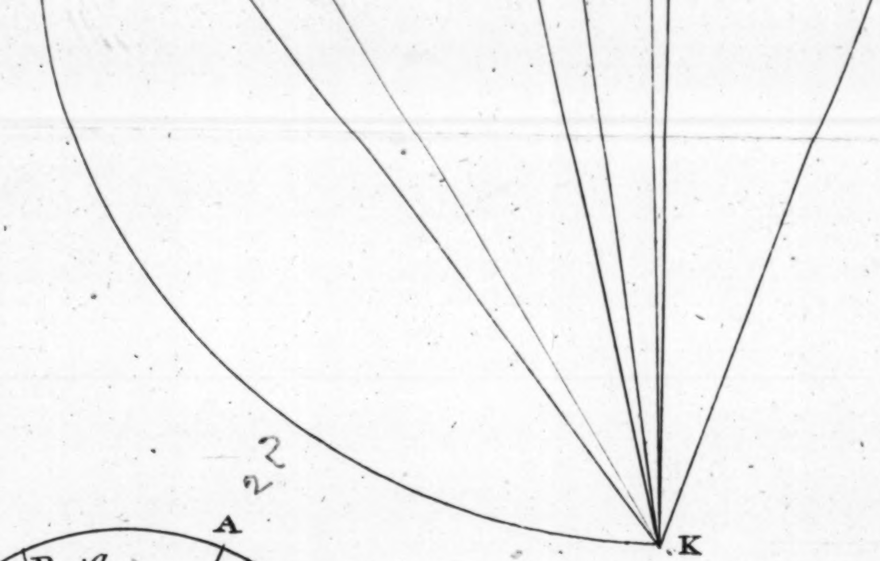

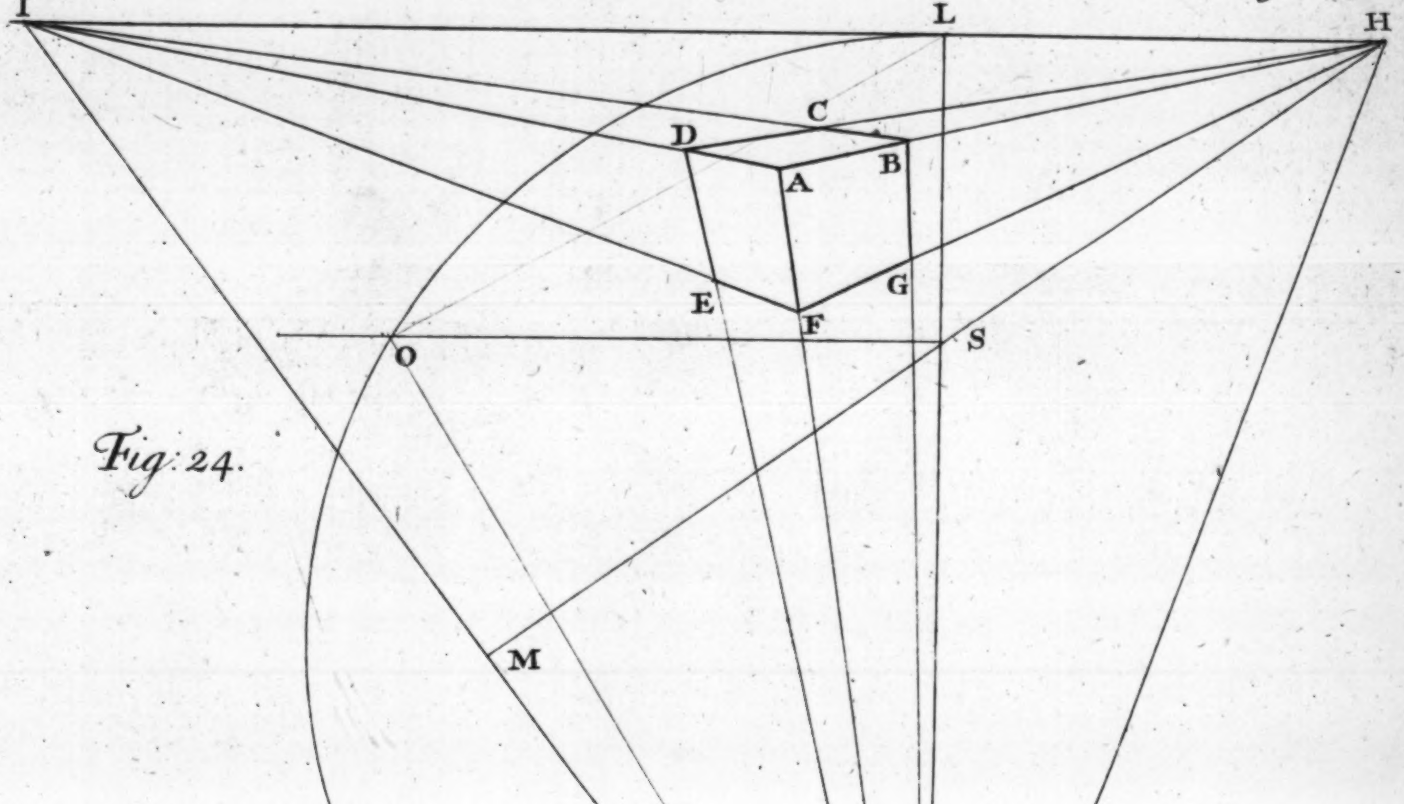

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